Force and Motion II: Vector Quantities

SPM Physics

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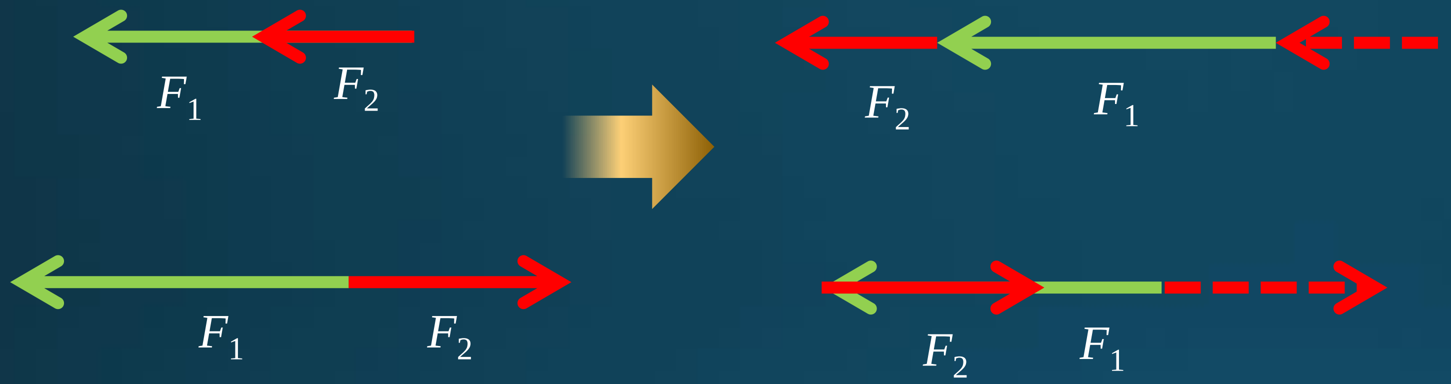
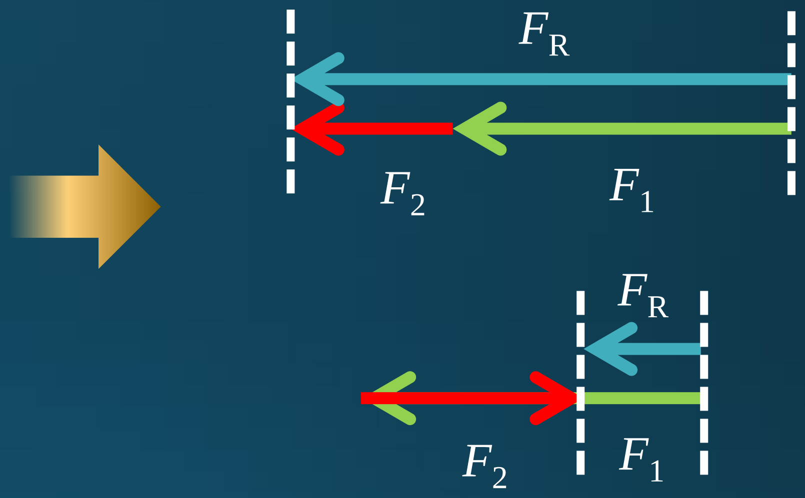
Addition of Forces

If two forces lie on the same line of action, then the forces can be added through the following steps:





Move the red arrow

Green arrow's tail joins with the **red arrow's head**

DO NOT CHANGE THE DIRECTION AND LENGTH OF THE RED ARROW!

Draw the blue arrow from the **green arrow's tail** to the **red arrow's head**

The blue arrow is the resultant force of F_1 and F_2.

This method also can be done if the green arrow moves while the red arrow does not move.

Based on the previous method, when there are two forces lie on the same line of action,

if they are directed at the **same direction**, then $F_R = F_1 + F_2$ (add up together)

If they are directed **opposite to each other**, then $F_R = F_1 - F_2$ (subtract each other)

Example:

As shown in Fig. 1, three horizontal forces are applied to a particle.

Determine the net force applied to the particle.

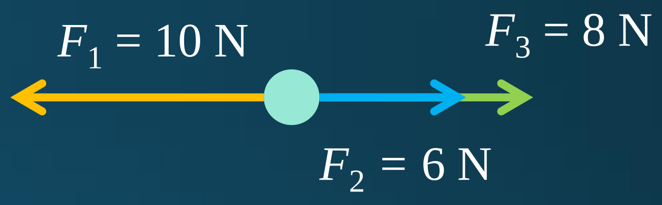


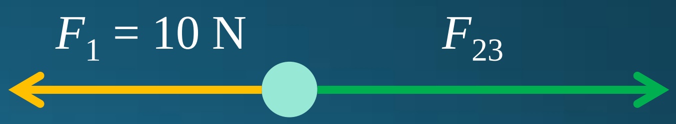
Fig. 1

Solution:

Find the forces directed at the same direction (i.e. F_2 & F_3) and add them up together,

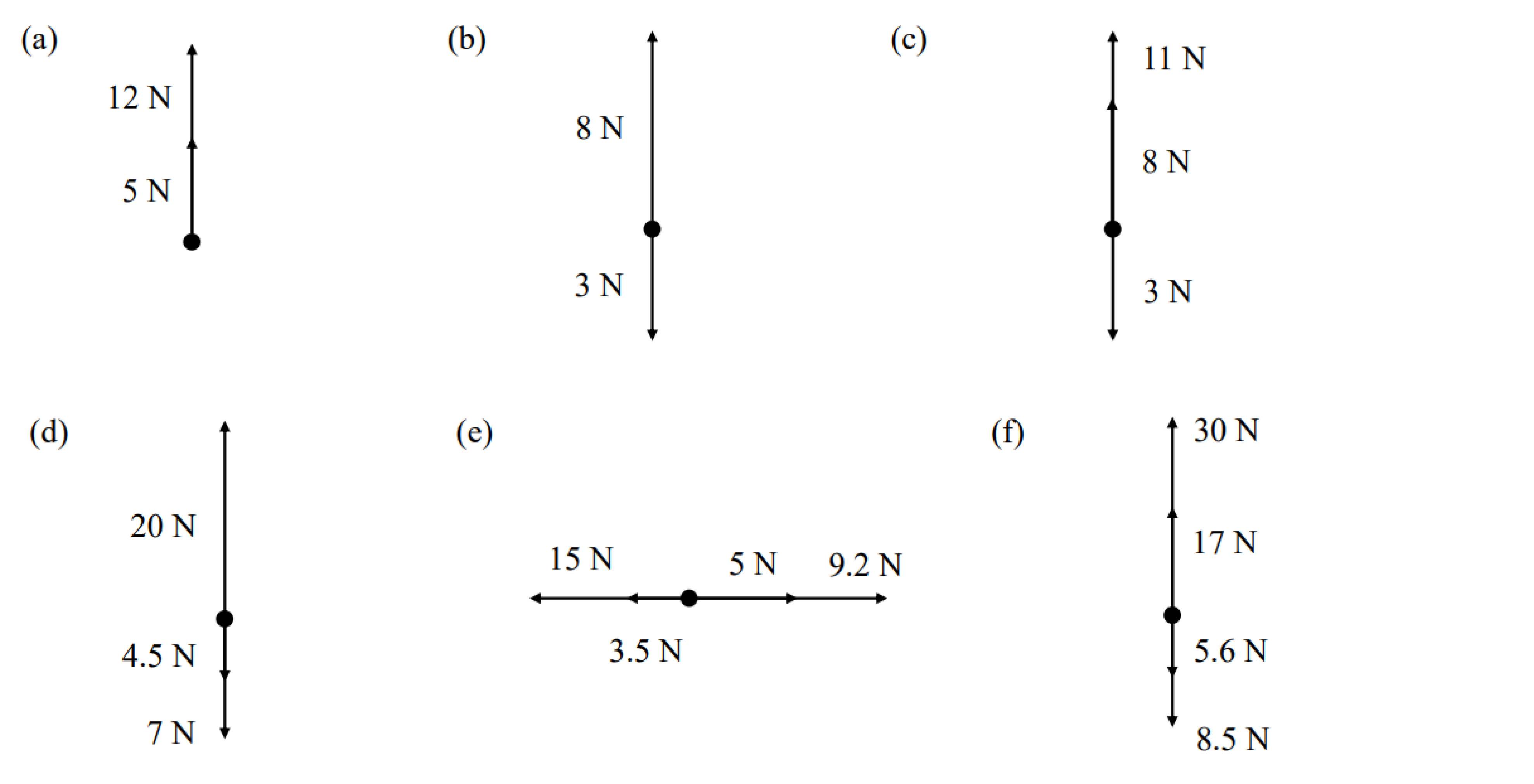
$F_{23} = F_2 + F_3 = 6 + 8 = 14$ N

Next, subtract the remaining two opposing forces (if any) with each other

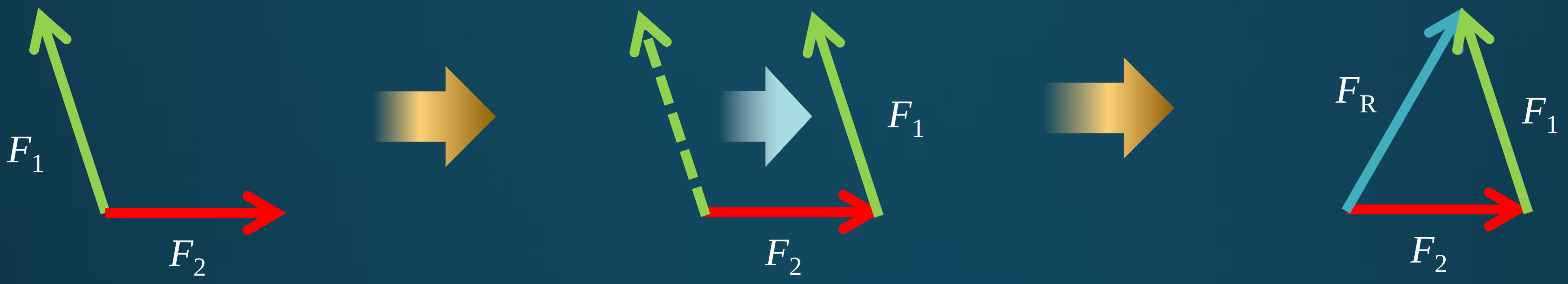


$F_R = F_{23} - F_1 = 14 - 10 = 4$ N (to the right)

For each questions, determine the net force applied to the particle.



- Triangle Method



Move the green arrow

The **green arrow's tail** joins with the **red arrow's head**

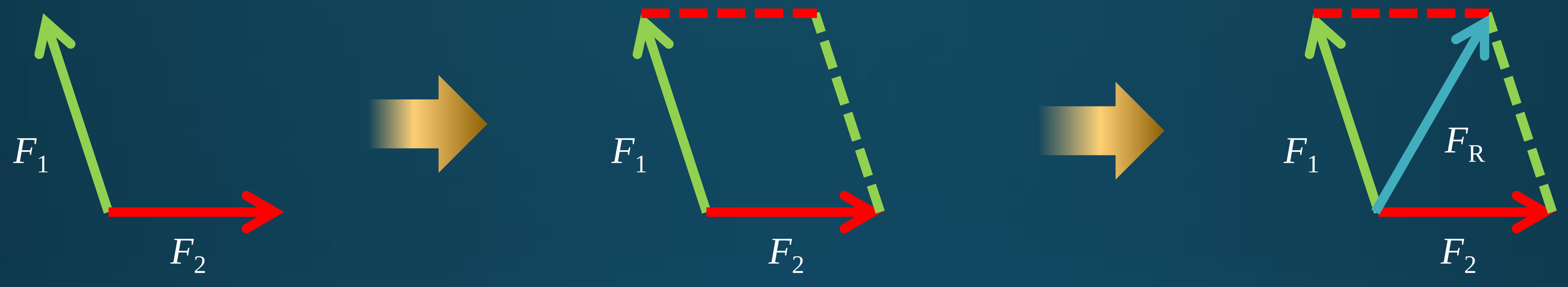
DO NOT CHANGE THE DIRECTION AND LENGTH OF THE GREEN ARROW!

Draw the blue arrow from the **red arrow's tail** to the **green arrow's head**

The blue arrow is the resultant force of F_1 and F_2.

This method also can be done if the red arrow moves while the green arrow does not move.

- Parallelogram Method



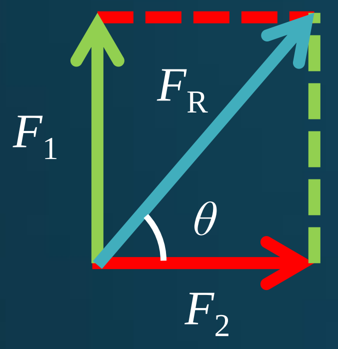
Draw out a parallelogram using the two forces as side lengths

Draw the blue arrow from the **red / green arrow's tail** to the **opposite vertex of the parallelogram**.

The blue arrow is the resultant force of F_1 and F_2.

This method also can be done if the red arrow moves while the green arrow does not move.

If these two forces are **perpendicular to each other**, then the length and direction of the resultant force can be calculated using Pythagoras theorem and normal trigonometric functions.



$$F_R = \sqrt{F_1^2 + F_2^2}$$

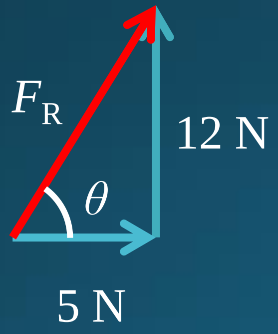
$$\sin\theta = \frac{F_1}{F_R} \qquad \cos\theta = \frac{F_2}{F_R} \qquad \tan\theta = \frac{F_1}{F_2}$$

Example:

As shown in Fig. 2, two forces are applied to a particle, and they are perpendicular to each other. Determine the resultant force applied to the particle.

Fig. 2

Solution:

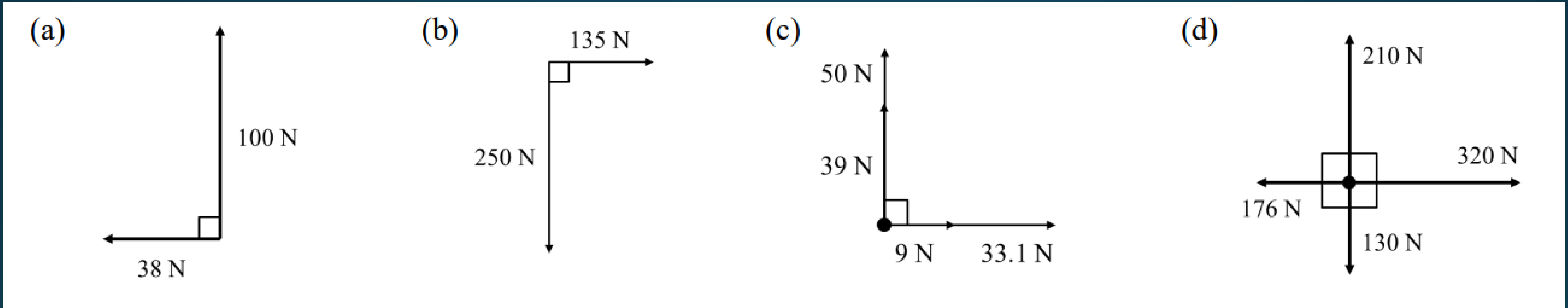


$$F_R = \sqrt{12^2 + 5^2} = 13 \text{ N}$$

$$\tan\theta = \frac{12}{5} \qquad \theta = 67.38^\circ$$

∴ The resultant force has a magnitude of 13 N and directed right upwards (makes an angle of 67.38° with the horizontal).

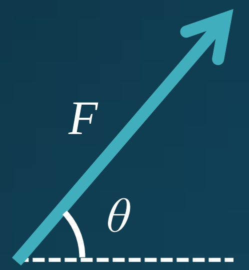
For each questions, determine the net force applied to the particle.

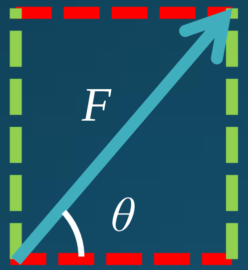


Resolution of Forces

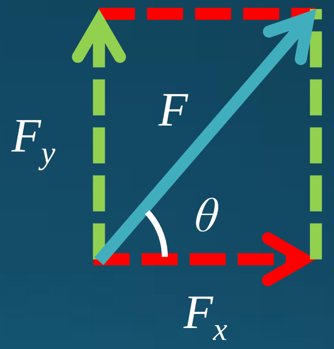
This method is the reversed version of force addition.

In most of the mechanics problems, it is preferred to resolve a force which is **neither vertical nor horizontal** into two components which are perpendicular to each other.





Create a rectangle with the blue arrow as one of its diagonal line.



$$\cos\theta = \frac{F_x}{F} \qquad \sin\theta = \frac{F_y}{F}$$

The horizontal and vertical arrows are the perpendicular components of force F.

Note: These two arrows' tails are joined at the blue arrow's tail!

Example:

A worker pulled a box placed on a ground with force $F = 100$ N which makes an angle of 30° with the horizontal, as shown in Fig. 3. Determine the horizontal and vertical component of the force F.

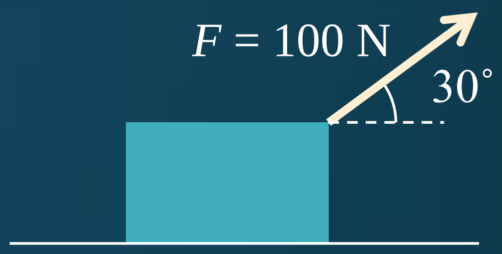
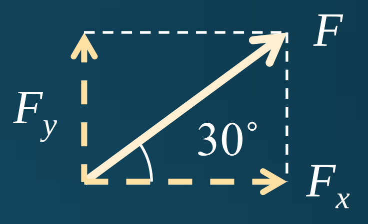


Fig. 3

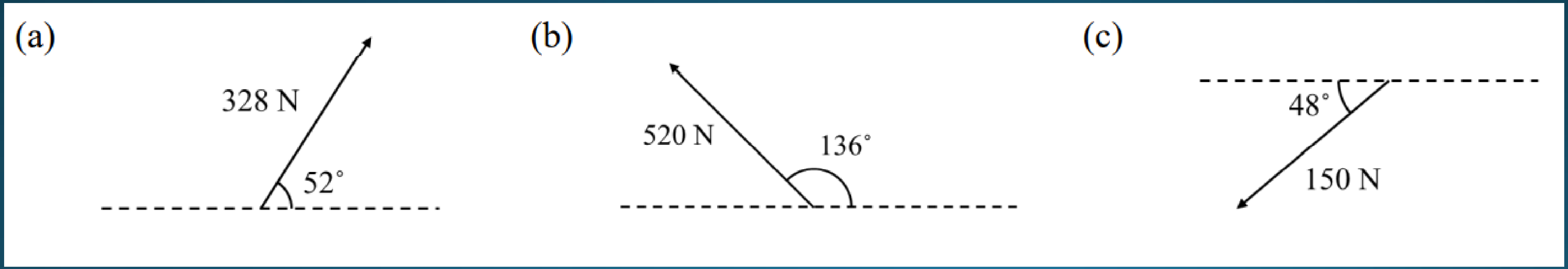
Solution:

Horizontal component of force $F = F_x = F\cos 30^\circ = 100\cos 30^\circ = 86.6$ N (right)

Vertical component of force $F = F_y = F\sin 30^\circ = 100\sin 30^\circ = 50$ N (upwards)



For each questions, determine the horizontal and vertical components of the force.



Forces in Equilibrium

If a particle is in equilibrium, the forces applied to it cancel out with each other.

In other words, the net horizontal and vertical force components applied to the particle are both zero, i.e.

$$\Sigma F_x = 0; \quad \Sigma F_y = 0$$

Question: If either of them is not equal to zero, how will the particle's motion behave?

Example:

As shown in Fig. 4, a ball with weight of 75 N is suspended by three strings. Determine the tensions in strings 1 and 2.

Solution:

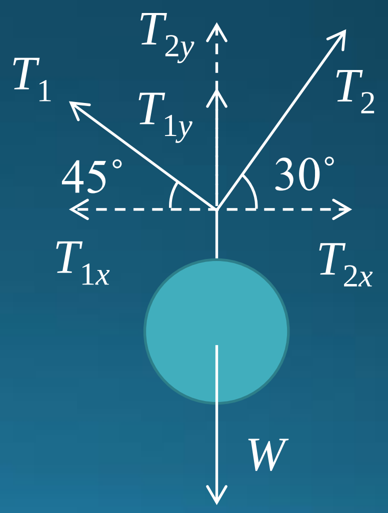
Divide T_1 and T_2 into x and y force components.

For x component, $\Sigma F_x = 0$;

$$T_{2x} - T_{1x} = 0$$

$$T_2 \cos 30^\circ - T_1 \cos 45^\circ = 0$$

$$T_2 = T_1 \frac{\cos 45^\circ}{\cos 30^\circ} \text{ ----- } (1)$$



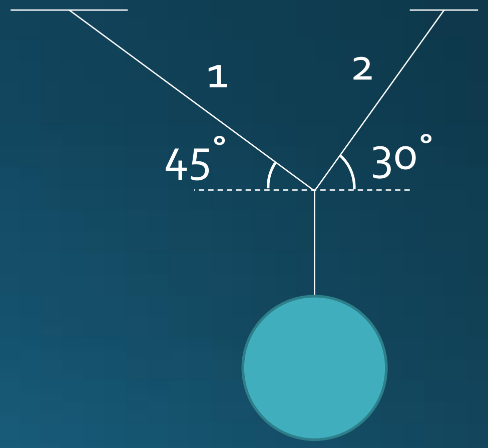


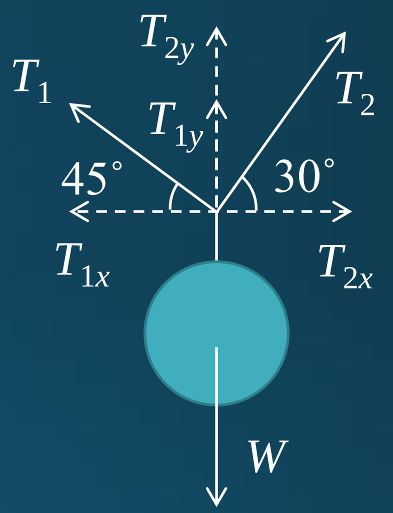
Fig. 4

For y component, $\Sigma F_y = 0$;

$$T_{1y} + T_{2y} - W = 0$$

$$T_1 \sin 30^\circ + T_2 \sin 45^\circ - 75 = 0$$

$$T_1 \sin 30^\circ + T_2 \sin 45^\circ = 75 \text{ -------- } (2)$$



Substitute (1) into (2), $T_1 \sin 30^\circ + T_2 \sin 45^\circ = 75$

$$T_1 \sin 30^\circ + T_1 \frac{\cos 45^\circ}{\cos 30^\circ} \sin 45^\circ = 75$$

$$T_1 = 69.62 \text{ N}$$

From (1), $T_2 = 69.42\left(\frac{\cos 45^\circ}{\cos 30^\circ}\right) = 56.84 \text{ N}$

Example:

As shown in Fig. 5, a 40 N block rests on a rough inclined plane which makes an angle of 60˚ with the horizontal. Determine the frictional force applied to the block.

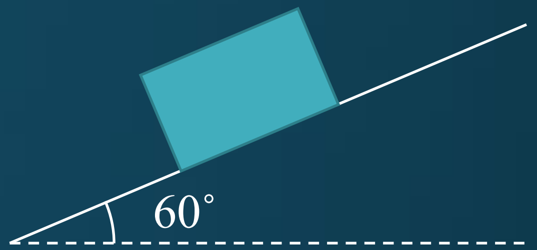


Fig. 5

Solution:

Take the **direction parallel to the inclined plane** as the **horizontal direction** and **its normal** as the **vertical direction**.

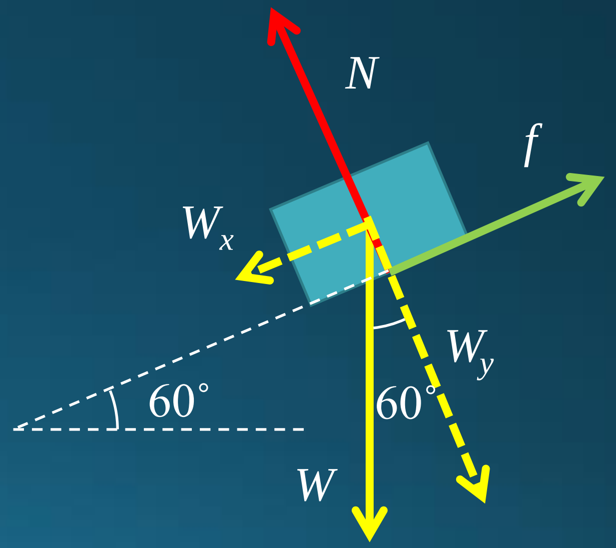
The weight W is neither horizontal nor vertical, hence it needs to be resolved.

As W_x and f are in **horizontal directions**, therefore f can be found from

$$\Sigma F_x = 0;$$

$$W_x - f = 0$$

$$f = W_x = W \sin 60^\circ = 40 \sin 60^\circ = 34.64 \text{ N}$$



1. As shown in Fig. 6, a 200 g picture frame is hung by two strings which are both connected to point P. Known that ∠PQR = 35˚ and ∠PRQ = 40˚. Determine the respective tensions in each strings.

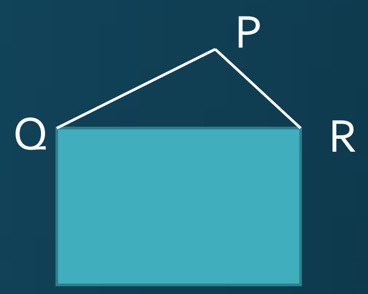


Fig. 6

2. As shown in Fig. 7, a 600 g block is pulled by force *F* parallel to the inclined plane. The block moves along the plane upwards with constant speed. If the magnitude of *F* is 10 N, determine the friction applied to the block.

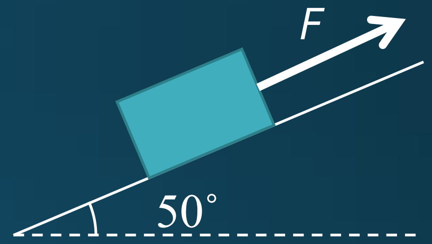


Fig. 7

3. As shown in Fig. 8, two cars B & C are pulling car A with magnitude of 30 N, both forces makes an angle of 37˚ with the horizontal. At the same time, car A is pulling to the left such that itself does not move. Determine the pulling force *F* applied by car A.

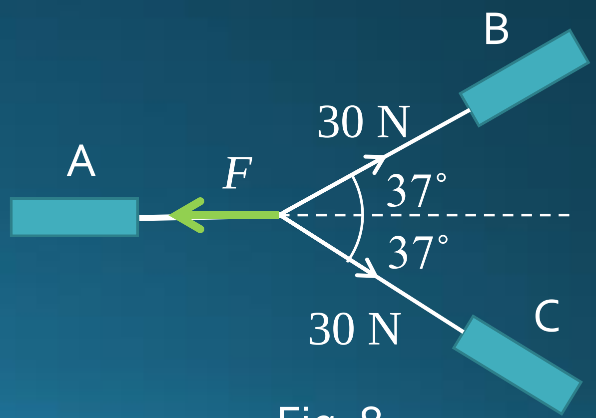


Fig. 8