

第十一章：流体力学 (2003年 - 2017年)

①  $M_{\text{顺}} = M_{\text{逆}}$   
 $m_x g l_x = m_y g l_y$   
 $V_x \rho l_x = V_y \rho l_y$   
 $\frac{V_x}{V_y} = \frac{l_x}{l_y}$   
 $\frac{V_x}{V_y} = \frac{4}{5}$

因浮力而产生的力矩

$M_{\text{顺}} = F_{\text{浮}x} l_x$        $M_{\text{逆}} = F_{\text{浮}y} l_y$   
 $= V_{\text{排}x} \rho_{\text{液}} g l_x$        $= V_{\text{排}y} \rho_{\text{液}} g l_y$   
 $= V_x \times 800 \times 9.8 \times 4$        $= V_y \times 1000 \times 9.8 \times 5$   
 $= 31360 V_x$        $= 49000 V_y$   
 $= 31360 \times \frac{4V_y}{5}$   
 $= 25088 V_y$  #

A

② B

③  $A_x V_x = n A_1 V_1$   
 $3 \times 30 = n \times 3 \times 10^{-7} \times 0.05$   
 $n = 6 \times 10^9$  #

D

④ D

⑤ 物体 A

$m_A g = V_{\text{排}} \rho_{\text{液}} g$   
 $\frac{1}{3} \rho_A g = \frac{2}{3} \rho_A \times 1000 g$   
 $\rho_A = \frac{2000}{3} \text{ kgm}^{-3}$

物体 B

$m_B g = V_{\text{排}} \rho_{\text{液}} g + \frac{1}{5} m_B g$   
 $\frac{4}{5} m_B g = V_{\text{排}} \rho_{\text{液}} g$   
 $\frac{4}{5} \rho_B = \rho_{\text{液}} \times 1000$   
 $\rho_B = 1250 \text{ kgm}^{-3}$

$\rho_A = \frac{2000}{3} \text{ kgm}^{-3}$   
 $\rho_B = 1250 \text{ kgm}^{-3}$  #

C

⑥  $P_0 + \frac{1}{2} \rho V_1^2 + h_p \rho g = P_0 + \frac{1}{2} \rho V_2^2 + h_B \rho g$   
 $P_0 + \frac{1}{2} \times 1000 \times 4^2 + h_p \times 1000 \times 9.8 = P_0 + \frac{1}{2} \times 1000 \times 2^2 + h_B \times 1000 \times 9.8$   
 $8000 + 9800 h_p = 2000 + 9800 h_B$   
 $9800 h_B - 9800 h_p = 8000 - 2000$   
 $9800 (h_B - h_p) = 6000$   
 $h_B - h_p = 0.61 \text{ m}$  #

B

⑦  $W = V_{\text{排}} \rho_{\text{液}} g$   
 $12 + 48 = (\frac{2}{3} V_B + V_{\text{全}}) \times 1000 \times 10$   
 $60 = (\frac{2}{3} \times \frac{m_B}{\rho_B} + \frac{m_{\text{全}}}{\rho_{\text{全}}}) \times 10000$   
 $60 = (\frac{2}{3} \times \frac{1.2}{\rho_B} + \frac{4.8}{7200}) \times 10000$   
 $6 \times 10^{-3} = (\frac{0.8}{\rho_B} + \frac{1}{1500})$   
 $\rho_B = 150 \text{ kgm}^{-3}$

B.

作答题

① (a)  $W_{\text{空}} - W_{\text{水}} = V_{\text{排}} \rho_{\text{液}} g$   
 $27.56 - 25.85 = V_{\text{排}} \times 1000 \times 9.8$   
 $V_{\text{排}} = 1.745 \times 10^{-4} \text{ m}^3$

$\rho = \frac{m}{V}$   
 $= \frac{27.56 - 25.85}{1.745 \times 10^{-4}}$   
 $= 1.61 \times 10^4 \text{ kgm}^{-3}$  #

(b)(i)  $P_0 + \frac{1}{2} \rho V_1^2 + h_p \rho g = P_0 + \frac{1}{2} \rho V_2^2 + h_B \rho g$   
 $P_0 + \frac{1}{2} \rho (0)^2 + 5 \times 1000 \times 9.8 = P_0 + \frac{1}{2} \times 1000 V_B^2 + 0$   
 $49000 = 500 V_B^2$   
 $V_B = 9.9 \text{ ms}^{-1}$

(ii) B点的流量 =  $A_B V_B$   
 $= 0.02 \times 9.9$   
 $= 0.198 \text{ m}^3 \text{ s}^{-1}$

$$\begin{aligned} \text{(iii)} \quad A_A V_A &= A_B V_B \\ 0.060 \times V_A &= 0.020 \times 9.9 \\ V_A &= 3.3 \text{ ms}^{-1} \end{aligned}$$

$$\begin{aligned} \text{动压强} &= \frac{1}{2} \rho V_A^2 \\ &= \frac{1}{2} \times 1000 \times 3.3^2 \\ &= 5445 \text{ Pa} \# \end{aligned}$$

$$\begin{aligned} T + \frac{m_{\text{油}}}{\rho_{\text{油}}} \rho_{\text{空}} g &= 14700 \\ T + \frac{1500}{19.3 \times 10^3} \times 1.2 \times 9.8 &= 14700 \\ T &= 14699 \# \end{aligned}$$

$$\begin{aligned} \text{(b) (i)} \quad \frac{F_A}{A_A} &= \frac{F_B}{A_B} \\ \frac{F_A}{\pi r_A^2} &= \frac{F_B}{\pi r_B^2} \\ \frac{15500}{16^2} &= \frac{F_B}{8^2} \\ F_B &= 3875 \text{ N} \# \end{aligned}$$

② (a) 浸在流体里的物体所受的浮力，其大小等于被物体所排开的流体的重量。利用阿基米德原理来制定轮船的载重线。

$$\begin{aligned} \text{(ii)} \quad p &= \frac{F}{A} \\ &= \frac{3875}{\pi (0.08)^2} \\ &= 1.93 \times 10^5 \text{ Pa} \# \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad p_1 + \frac{1}{2} \rho v_1^2 + h_1 \rho g &= p_2 + \frac{1}{2} \rho v_2^2 + h_2 \rho g \\ p_1, p_2 &= \text{流体的静压强} \\ v_1, v_2 &= \text{流速} \\ h_1, h_2 &= \text{流体的高度} \\ \rho &= \text{流体的密度} \end{aligned}$$

$$\begin{aligned} \text{(4)} \quad m_{\text{物}} g &= V_{\text{排}} \rho_{\text{液}} g \\ &= (0.15)^3 \times \frac{3}{5} \times 1000 \times 9.8 \\ &= 19.845 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad A_A V_A &= A_B V_B \\ \pi r_A^2 V_A &= \pi r_B^2 V_B \\ 0.3^2 \times 0.32 &= 0.15^2 V_B \\ V_B &= 1.28 \text{ ms}^{-1} \end{aligned}$$

$$\begin{aligned} m_{\text{物}} g &= V_{\text{排}} \rho_{\text{水}} g + V_{\text{排}} \rho_{\text{油}} g \\ 19.845 &= (h_{\text{水}} \times 0.15^2 \times 1000 \times 9.8) + \\ &\quad (0.07 \times 0.15^2 \times 800 \times 9.8) \end{aligned}$$

$$h_{\text{水}} = 0.034$$

$$\begin{aligned} h &= 0.15 - 0.07 - 0.034 \\ &= 0.046 \text{ m} \# \end{aligned}$$

$$\begin{aligned} p_A + \frac{1}{2} \rho v_A^2 + h_A \rho g &= p_B + \frac{1}{2} \rho v_B^2 + h_B \rho g \\ 120 \times 1000 + \frac{1}{2} \times 1000 \times 0.32^2 &= p_B + \frac{1}{2} \times 1000 \times 1.28^2 \\ &\quad + 2 \times 1000 \times 9.8 \\ p_B &= 99.632 \text{ kPa} \# \end{aligned}$$

$$\begin{aligned} \text{(3) (a)} \quad T + F_{\text{浮}} &= mg \\ T + V_{\text{排}} \rho_{\text{液}} g &= 1500 \times 9.8 \\ T + \frac{m_{\text{油}}}{\rho_{\text{油}}} \rho_{\text{液}} g &= 14700 \\ T + \frac{1500}{19.3 \times 10^3} \times 1.03 \times 10^3 \times 9.8 &= 14700 \\ T &= 13915 \text{ N} \# \end{aligned}$$