第八章：动量守恒定律（2005年－2016年）

①
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$mv + M(0) = mv_1 + Mv_2$$
$$mv = mv_1 + Mv_2$$
$$mv = mv_1 + 8mv_2$$
$$v = v_1 + 8v_2 \quad ①$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$$
$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(8m)v_2^2$$
$$v^2 = v_1^2 + 8v_2^2 \quad ②$$
$$(v_1 + 8v_2)^2 = v_1^2 + 8v_2^2$$
$$v_1^2 + 16v_1v_2 + 64v_2^2 = v_1^2 + 8v_2^2$$
$$16v_1v_2 = -56v_2^2$$
$$16v_1 = -56v_2$$
$$v_1 = -3.5v_2 \quad ③$$

$$\frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{v^2 - v_1^2}{v^2}$$
$$= \frac{8v_2^2}{(v_1 + 8v_2)^2}$$
$$= \frac{8v_2^2}{(-3.5v_2 + 8v_2)^2}$$
$$\frac{8v_2^2}{(4.5v_2)^2}$$
$$= 0.395 \#$$

A

②弹性碰撞，动能是守恒。

A

③
$$0 = m_1v_1 + m_2v_2$$
$$0 = -v_1 + 4v_2$$

$$\frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2} = \frac{1 \times v_1^2}{4 \times v_2^2}$$
$$= \frac{v_1^2}{4(\frac{v_1}{4})^2}$$
$$= 4 \#$$

D

④ C

⑤
$$v_B - v_A = 1 \quad ①$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$12 \times 10 + 8 \times 6 = 12v_A + 8v_B$$
$$168 = 12v_A + 8(1 + v_A)$$
$$168 = 12v_A + 8 + 8v_A$$
$$20v_A = 160$$
$$v_A = 8 \#$$
$$v_B = 9 \#$$

B

作答题：

①
$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
$$0.02 \times u + 4.98 \times 0 = (0.02 + 4.98)v$$
$$0.02u = 5v \quad ①$$

$$\frac{1}{2}mv^2 = mgh$$
$$\frac{1}{2}(0.02 + 4.98)v^2 = (0.02 + 4.98)9.8 \times 0.1$$
$$\frac{5}{2}v^2 = 4.9$$
$$v = 1.4$$

$$0.02u = 5 \times 1.4$$
$$u = 350\,ms^{-1} \#$$

② (a) $mgh = \frac{1}{2}mu^2$ ①

$9.8 \times 4 = \frac{1}{2}u^2$ ②

$u = 8.85\ ms^{-1}$ # ③

$m_1u_1 + m_2u_2 = (m_1+m_2)V$ ①

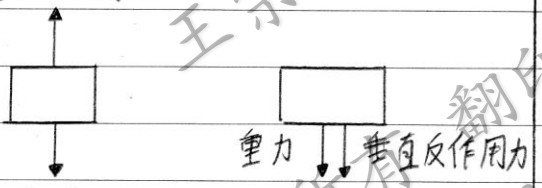
$1 \times 8.85 + 1 \times 0 = (1+1)V$ ③

$8.85 = 2V$ ③

$V = 4.43\ ms^{-1}$ # ③

(b) P点 Q点

垂直反作用力



(c) P点的垂直反作用力

$= \frac{mv^2}{r} + mg$ ①

$= \frac{1 \times 4.43^2}{0.3} + 1 \times 9.8$ ③

$= 95.2\ N$ # ③

$\frac{1}{2}mu^2 = mgh + \frac{1}{2}mv^2$ ①

$\frac{1}{2} \times 4.43^2 = 9.8 \times 0.6 + \frac{1}{2}V^2$ ③

$V = 2.80\ ms^{-1}$ ③

Q点的垂直反作用力

$= \frac{mv^2}{r} - mg$ ①

$= \frac{1 \times 2.8^2}{0.3} - 1 \times 9.8$ ③

$= 16.4\ N$ # ③

③ (a) 小船靠岸边，若没有固定，当人在船上向前走打算离船上岸时，会感觉船都向反移去，好像故意不让人登岸；而如果人返过身来往回原位走，小船又向岸边移去，像有意做弄人一样，也可以解释为人和船发生了相互作用，人向后蹬船，船给人一个向前的作用力，人得到了向前的动量，船同时也得到向后的动量，以保持相互的总动量仍为零。人向后走时，船又向前移去，两者的动量之和仍为零（忽略水对船的摩擦阻力）

(b) $m_1u_1 + m_2u_2 = (m_1+m_2)V$

$3 \times 1.2 + 2 \times 0 = (3+2)V$

$3.6 = 5V$

$V = 0.72\ ms^{-1}$ #

(c) $m_Au_A + m_Bu_B = m_AV_A + m_BV_B$

$2 \times 3 + 2 \times 0 = 2V_A + 2V_B$

$6 = 2V_A + 2V_B$

$V_A + V_B = 3$

$\frac{1}{2}m_Au_A^2 = \frac{1}{2}m_AV_A^2 + \frac{1}{2}m_BV_B^2$

$\frac{1}{2} \times 2 \times 3^2 = \frac{1}{2} \times 2 \times V_A^2 + \frac{1}{2} \times 2 \times V_B^2$

$9 = V_A^2 + V_B^2$

$9 = (3-V_B)^2 + V_B^2$

$9 = 9 - 6V_B + V_B^2 + V_B^2$

$0 = 2V_B^2 - 6V_B$

$0 = V_B(V_B - 3)$

$V_B = 0$（不接受） $V_B = 3ms^{-1}$

$V_B = 3ms^{-1}$; $V_A = 0ms^{-1}$ #

④ (a) 水平方向

$m_1u_1 + m_2u_2 = (m_1+m_2)V$

$8 \times 6 + 5 \times 8\cos 45 = (8+5)V$

$V = 5.87\ ms^{-1}$

竖直方向.

$m_1u_1 + m_2u_2 = (m_1+m_2)V$

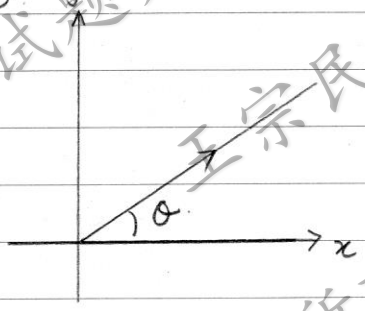
$8\times0 + 5\times8\sin45 = (8+5)V$

$V = 2.176\,ms^{-1}$

$V = \sqrt{5.87^2+2.176^2}$

$= 6.26\,ms^{-1}$ #

(b)



$\tan\theta = \frac{2.176}{5.87}$

$\theta = 20.34°$ #

(c) $\frac{\frac{1}{2}(m_A+m_B)V^2}{\frac{1}{2}m_Au_A^2+\frac{1}{2}m_Bv_B^2}\times100\%$

$= \frac{(8+5)\times6.26^2}{8\times6^2+5\times8^2}\times100\%$

$= 83.79\%$

总动能的损失百分率 $= 100\% - 83.79\%$

$= 16.21\%$ #

⑤ (a) 如果两物体所组成的系统不受外力或所受合外力为零，则两物体在碰撞前后的总动量保持不变。

(b)(i) $m_1u_1 = (m_1+m_2+m_3)V$

$0.05\times100 = (0.05+1+0.8)V$

$V = 2.7\,ms^{-1}$ #

(ii) $\frac{1}{2}m_1u^2 = \frac{1}{2}(m_1+m_2+m_3)V + (m_1+m_2)gh + E_{loss}$

$\frac{1}{2}\times0.05\times100^2 = \frac{1}{2}(0.05+1+0.8)2.7^2 + (0.05+1)\times9.8\times0.6 + E_{loss}$

$E_{loss} = 237.08\,J$ #

⑥ (a) 牛顿第三运动定律，反作用力的大小等于作用力的大小且方向相反.

$F_1 = -F_2$

$\frac{m_1(V_1-u_1)}{t} = \frac{-m_2(V_2-u_2)}{t}$

$m_1V_1 - m_1u_1 = -m_2V_2 + m_2u_2$

$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$ #

(b)(i) 在弹性碰撞中，总动量和总动能都守恒。

(ii) 在非弹性碰撞中，只有总动量守恒，总动能会损失。

(c)(i) $m_1u_1 = (m_1+m_2+m_3)V$

$0.1\times100 = (0.1+1.9+8)V$

$V = 1\,ms^{-1}$

$u = 1$ $\quad v^2 = u^2 + 2as$

$s = ?$ $\quad 0 = 1^2 + 2(-0.4\times9.8)s$

$v = 0$ $\quad s = 0.128$ #

$a = -0.4\times9.8$

$\frac{1}{2}(m_1+m_2+m_3)V =$

⑦ (a) 第 2 秒 #

(b) (i) 碰撞前：P球以匀速前进，而Q球则静止。

碰撞后：P球反向匀速运动，而Q球则沿P球原来的方向匀速运动。

(ii)

	碰撞前	碰撞后
P球	$\frac{6}{2} = 3 ms^{-1}$	$\frac{-8}{4} = -2 ms^{-1}$
Q球	$0 ms^{-1}$	$\frac{4}{4} = 1 ms^{-1}$

(c) 碰撞前的总动能

$= \frac{1}{2} m_p u^2$

$= \frac{1}{2} \times 1 \times 3^2$

$= 4.5 J$ #

碰撞后的总动能

$= \frac{1}{2} m_p v_p^2 + \frac{1}{2} m_Q v_Q^2$

$= \frac{1}{2} \times 1 \times (-2)^2 + \frac{1}{2} \times 5 \times 1^2$

$= 4.5 J$ #

总动能守恒

碰撞前的总动量

$= m_p u$

$= 1 \times 3$

$= 3 kgms^{-1}$

碰撞后的总动量

$= m_p v_p + m_Q v_Q$

$= 1 \times (-2) + 5(1)$

$= 3 kgms^{-1}$

总动量守恒

⑧ (a) $v = \sqrt{gr}$

$= \sqrt{9.8 \times 0.5}$

$= 2.21 ms^{-1}$ #

(b) $\frac{1}{2} m v_1^2 + mgh = \frac{1}{2} m v^2$

$\frac{1}{2} \times 2.21^2 + 9.8 \times 1 = \frac{1}{2} v^2$

$v = 4.95 ms^{-1}$ #

(c) $m_1 u = (m_1 + m_2) V$

$0.1 u = (0.1 + 0.5) \times 4.95$

$u = 29.7 ms^{-1}$ #

(d) 碰撞而损失的能量

$= \frac{1}{2} m_1 u^2 - \frac{1}{2} (m_1 + m_2) v^2$

$= \frac{1}{2} \times 0.1 \times 29.7^2 - \frac{1}{2} (0.1 + 0.5) \times 4.95^2$

$= 36.75 J$ #

⑨ (a) (i) 动量改变量

$= mv - mu$

$= 1.2 \times (-1) - 1.2 \times 4$

$= -6 kgms^{-1}$ #

(ii) $F = \frac{mv - mu}{t}$

$= \frac{-6}{0.08}$

$= -75 N$ #

(iii) $m_1 u_1 = m_1 v_1 + m_2 v_2$

$1.2 \times 4 = 1.2 \times (-1) + 3.6 v_2$

$v_2 = 1.67 ms^{-1}$ #

(b) $m_1 u_c = (m_1 + m_2) V$

$4 u_c = (4 + 20) 1$

$u_c = 6 ms^{-1}$ #

$\frac{1}{2}m_1u_c^2 = \frac{1}{2}(m_1+m_2)v^2 + mg\mu L$

$\frac{1}{2}\times 4\times 6^2 = \frac{1}{2}(4+20)1^2 + 4\times 10\times 0.6\times L$

$72 = 12 + 24L$

$L = 2.5\,m$ #

(10) (a) $mu = m_1v_1 + m_2v_2$

$m\times 20\cos 60 = \frac{2}{3}\times m\times 0 + \frac{1}{3}\times m\times v_2$

$v_2 = 30\,ms^{-1}$ #

(b) 爆开前

• 竖直方向：

$u = 20\sin 60$ $\quad v = u + at$

$v = 0$ $\quad 0 = 20\sin 60 - 9.8t$

$a = -9.8$ $\quad t = 1.767s$

$t = ?$

• 水平方向：

$s = ut$

$= 20\cos 60 \times 1.767$

$= 17.67m$

爆开后

水平方向：

$s = ut$

$= 30 \times 1.767$

$= 53.01m$

∴ 水平距离 $= 53.01m + 17.67m$

$= 70.68m$ #

(11) (a) 设 A 点为转轴

$\Sigma M = 0$

$-20\times 0.5 + C\times 0.4 - 4\times 1 = 0$

$C = 35N$ #

(b)(i) $\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + mgh$

$\frac{1}{2}v_1^2 = \frac{1}{2}v_2^2 + 10\times(0.4)$

$v_1^2 = v_2^2 + 8$ #

(ii) 设 A 点为转轴

$\Sigma M = 0$

$(-20\times 0.5) + (T\times 1) = 0$

$T = 10N$ #

$T = \frac{mv_2^2}{r} - mg$

$10 = \frac{0.4v_2^2}{0.2} - 0.4\times 10$

$v_2 = 2.646\,ms^{-1}$ #

$v_1^2 = 7 + 8$

$v_1 = 3.87\,ms^{-1}$ #

(iii) $m_2u = m_2v_2 + m_1v_1$

$0.1\times v_0 = 0.1\times(-0.6v_0) + (0.4\times 3.$

$0.1v_0 = -0.06v_0 + 1.548$

$v_0 = 9.675\,ms^{-1}$ #

(12) (a) 摩擦力 $= mg\mu$

$= 2\times 10\times 0.2$

$= 4N$ #

(b) $s = 1$ $\quad v^2 = u^2 + 2as$

$a = \frac{4}{2}$ $\quad 0^2 = u_A^2 + 2(2)(1)$

$v = 0$ $\quad u_B = 2\,ms^{-1}$ #

$u_B = ?$

(c) $m_Au_A = m_Av_A + m_Bv_B$

$1u_A = 1\times\frac{-u_A}{3} + 2(2)$

$u_A = 3\,ms^{-1}$ #

(d) $T = \frac{mv^2}{r} + mg$

$= \frac{1 \times 3^2}{0.5} + (1 \times 10)$

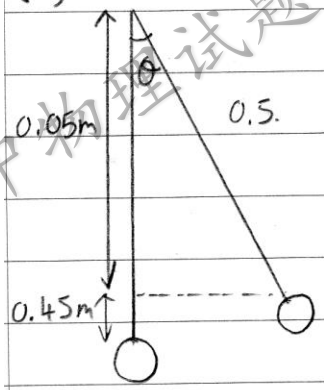
$= 28\ N$ #

(e) $\frac{1}{2}mv^2 = mgh$.

$\frac{1}{2} \times 3^2 = 10h$.

$h = 0.45\ m$ #

(d)



$0.05 = 0.5 \cos\theta$

$\theta = 84.26°$ #