第七章 因式分解

1. 共同公因

例子

$9ac + 12ab = 3a(3c+4b)$

$3 \mid 9ac+12ab$

$a \mid 3ac+4ab$

$3c+4b$

2. 平方差公式

$a^2 - b^2 = (a+b)(a-b)$

例子

$9-x^2 = (3+x)(3-x)$

3.完全平方公式

$a^2+2ab+b^2 = (a+b)^2$

$a^2-2ab+b^2 = (a-b)^2$

例子

$x^2-4x+4 = (x+2)(x-2)$

$A^2+b^2+c^2+2ab+2bc+2ca = (a+b+c)^2$

3.立方差公式

$x^3+3x^2y+3xy^2+y^2 = (x+y)^3$

$x^3-3x^2y+3xy^2-y^3 = (x-y)^3$

例子 $x^3+6x^2y=3xy^2+8y^2 = (x+2y)^3$

$x^3 - y^2 = (x-y)(x^2+y^2+xy)$

$x^3+y^3 = (x+y)(x^2+y^2-xy)$

例子

$8x^3-y^3$

$= (2x-y)(2x^2+y^2+2(2x)(y))$

$=(2x-y)(4x^2+y^2+2xy)$

交叉相乘法

例子 $5x^2 +13x -6$

$5x$	-2	$-2x$
$1x$	3	$15x$
$5x^2$	-6	$13x$

$(5x-2)(x+3)$

分组分解

$ax+ay+bx+by$

$=a(x+y)+b(x+y)$

$=(a+b)(x+y)$

HCF/LCM

例子

x^2 ,$2x$,x^3 的 HCF

$x \mid x^2, 2x, x^3$

$x, 2, x^2$

HCF=x

练习

1	m^4-3m^2	2	$2x^2+x-15$
3	$8x^2-4x$	4	$6xy+10x^2y$
5	$h^2+8h+15$	6	x^2-9
7	$\frac{P^2-1}{(1+2p)^2-(p+2)^2}$	8	$1291^2-1291\times1281$
9	$ab+bc-dc-da$	10	$998^2-4=$
11	$a+b=2,ab+\frac{a^2+b^2}{2}=$	12	$2x^2+4$
13	$\sqrt{62.5^2-58\cdot5^2}$	14	$\frac{a}{a^2-16}-\frac{a-8}{16-a^2}=$
15	$\frac{3(4p^2+9)(2p+3)}{21(16p^4-81)}=$	16	$3x^2-5x-12$
17	$2x^2+6$	18	$4x^2-1$
19	$a+b=1,a=m+\frac{a^2-b^2}{2},m=?$	20	x^2-1
21	$10xy-25x^2-y^2=$	22	$\frac{a^2-a-ab+b}{(a-b)(1-a)}=$
23	$x^2-y^2+6x-6y=$	24	$x=\frac{a+2b}{2},y=\frac{a-2b}{2},x^2-y^2=$
25	$\frac{2}{x-5}+\frac{3}{x+5}=\frac{5}{x^2-25},x=?$	26	$a^2+b^2=2,ab=3,(a+b)^4=$
27	$\frac{y^2+y-6}{y+1}\div\frac{y+3}{2y^2+y-1}=$	28	$\frac{a^2-ab-ac+bc}{2a^2-ab-b^2}$
29	$(3a+2)^2-3(3a+2)(a-2)=$	30	$\frac{6x}{x-y}-\frac{x+5y}{x-y}=$
31	$(x+3)^2-(x+3)^2=$	32	$\frac{(4ab^2)(4cd^2)}{2cb^2}$
33	$\frac{x}{x-2}-\frac{3}{x+1}=\frac{9}{x^2-x-2}$	34	$\frac{6ab}{5c}\div\frac{3a}{10c^2}$
35	$\frac{a^2-9}{a^2-6a+9}\times\frac{a^2-3a}{a^2}=$	36	$\frac{3x+1}{3x-1}-\frac{2x+1}{x-1}=0$
37	$\frac{ab^2-a^2b}{a-b}$	38	$\frac{x}{x-2}+\frac{4}{x-3}=\frac{2}{(3-x)(x-2)}$
39	$\frac{x-1}{x^2-5x+6}+\frac{x}{x-2}=\frac{2}{x-3}$	40	$\frac{2}{3x-1}+\frac{1}{x-1}-\frac{7}{3x^2-4x+1}=0$

分组分解

1	$(3x+2y)^3-6x-4y=$	2	$x^2-1+y(x+1)=$
3	$12x^3+2x^2-30x-5=$	4	$ax-bx+a^2-b^2=$
5	$4xy+6-x-24y=$	6	$(a+3b)(2a-b)-(a+3b)^2=$
7	$12x^2u+3x^2v+28yu+7yv=$	8	$(a-b)(a+2b)-2a+2=$
9	$42mc+36md-7n^2c-6n^2d=$	10	$ab+bc-da-dc=$
11	$56xw+49xk^2-24yw-21yk^2=$	12	$6p(3x+2y)-(3p+2q)(3x+2y)=$
13	$24r^3-64r^2-21r+56=$	14	$a^2-b^2+a+b=$
15	$x(x-1)-x+1=$	16	$x^2-y^2+6x-6y=$
17	$a^2-ab+5a-5b=$	18	$25a^2-4b^2+5a+2b=$
19	$ac-bc-a^2+2ab-b^2=$	20	$4q+4p-q^2-pq=$

LCM/HCF

1	x^2+3x-4,x^2-4x+3	2	$(x-1),x^2-2x+1,2x$
3	$p^2-q^2,2(p+q),3(p-q)$	4	$a^2+b^2+2ab,(a+b)^3,a^2(a+b)-b^2(a+b)$
5	$21a^2b,9abc$	6	xy,x^3y^2
7	$a^2-ab-2b^2,a^2+3ab+2b^2$	8	$x^2+9x-10,x^2-14x+48$
9	$x^2-2x-15,x^2-9x+18,x^2-7x-30$	10	$2(x+y),3(x+y)^2,4(x+y)^3$