

因式分解

I. Common Factor

Eg. Factorise $9ac + 12ab$

$$\begin{array}{r|l} 3 & 9ac \quad 12ab \\ a & 3ac \quad 4ab \\ \hline & 3c \quad 4b \end{array}$$

Therefore, $9ac + 12ab = 3a(3c + 4b)$

II. Formulae

$$a^2 - b^2 = (a + b)(a - b)$$

Eg. $\sqrt{9} = 3 \rightarrow a$

$$\sqrt{x^2} = x \rightarrow b$$

$$9 - x^2 = (3 + x)(3 - x)$$

III. Perfect square formulae

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$$

IV. Perfect cube formulae

$$x^3 + 3x^2y + 3xy^2 + y^3 = (x + y)^3$$

$$x^3 - 3x^2y + 3xy^2 - y^3 = (x - y)^3$$

Eg. $x^3 + 6x^2y + 3xy^2 + 8y^3$

$$\sqrt[3]{x^3} = x$$

$$\sqrt[3]{8y^3} = 2y$$

$$x^3 + 6x^2y + 3xy^2 + 8y^3 = (x + 2y)^3$$

Eg. $27x^3 - 27x^2y + 9xy^2 - y^3$

$$\sqrt[3]{27x^3} = 3x$$

$$\sqrt[3]{y^3} = y$$

$$27x^3 - 27x^2y + 9xy^2 - y^3 = (3x - y)^3$$

$$x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$$

$$x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$$

Eg. $8x^3 - y^3$

$$\sqrt[3]{8x^3} = 2x$$

$$\sqrt[3]{y^3} = y$$

$$8x^3 - y^3 = (2x - y)(4x^2 + y^2 + 2xy)$$

V. Cross multiplication

Factorise $x^2 + x - 12 = (x - 3)(x + 4)$

| | | |
|-------|--------|--------|
| x | $- 3$ | $- 3x$ |
| x | 4 | $+ 4x$ |
| x^2 | $- 12$ | x |

Exercise

Section 1 :Factorization

- | | | |
|--------------------------------------|------------------------------------|----------------------|
| 1. $xz + z^2$ | 14. $x^3 - 9x^2y + 27xy^2 - 27y^3$ | 28. $a^2 - 9a + 14$ |
| 2. $10xy - 5x^2$ | 15. $125x^3 - 8y^3$ | 29. $x^2 + 8x + 12$ |
| 3. $x^3y - xy^2$ | 16. $x^2 + 3x + 2$ | 30. $p^2 - 8p + 15$ |
| 4. $abc + b^2c$ | 17. $x^2 + x - 2$ | 31. $b^2 - 16b + 15$ |
| 5. $12a - 14a^3$ | 18. $x^2 - 4x + 3$ | 32. $e^2 - 7e + 12$ |
| 6. $121 - y^2$ | 19. $x^2 + 4x + 3$ | 33. $a^2 - a - 20$ |
| 7. $x^2y^2 - 36y^2$ | 20. $y^2 - 4y - 5$ | 34. $x^2 - 9x + 18$ |
| 8. $\frac{a^2}{25} - \frac{4b^2}{9}$ | 21. $b^2 - 6b - 7$ | 35. $x^2 + 4x - 21$ |
| 9. $\frac{x^2}{4} - \frac{y^2}{9}$ | 22. $x^2 - x - 6$ | 36. $y^2 - 7y - 18$ |
| 10. $49x^4y^4 - 25z^4$ | 23. $a^2 + 6a + 8$ | 37. $n^2 - 5n - 24$ |
| 11. $x^2 - 2x + 1$ | 24. $m^2 + 9m + 8$ | 38. $m^2 + 11m + 28$ |
| 12. $8x^3 + 12x^2y + 16xy^2 + y^3$ | 25. $m^2 - 7m - 8$ | 39. $y^2 + 7y + 10$ |
| 13. $x^3 - 27y^3$ | 26. $b^2 - 3b - 10$ | 40. $x^2 - x - 42$ |
| | 27. $c^2 - c - 12$ | 41. $y^2 + 10y + 24$ |
| | | 42. $x^2 + 5x - 36$ |

$$43.p^2 - 14p + 24$$

$$44.n^2 + 12n + 32$$

$$45.d^2 + 18d + 45$$

$$46.2x^2 + 5x + 3$$

$$47.2x^2 - 7x + 3$$

$$48.3x^2 + 7x + 2$$

$$49.3k^2 + k - 2$$

$$50.2a^2 + a - 1$$

$$51.3y^2 - 2y - 1$$

$$52.3m^2 - 5m - 2$$

$$53.5y^2 - 4y - 1$$

$$54.5m^2 + 2m - 3$$

$$55.2a^2 + 11a + 5$$

$$56.2y^2 + y - 6$$

$$57.3a^2 - 7a - 6$$

$$58.2p^2 - 19p + 9$$

$$59.3m^2 - 2m - 8$$

$$60.5x^2 - 23x - 10$$

$$61.3a^2 + 14a + 15$$

$$62.3y^2 + 10y - 8$$

$$63.2p^2 + 7p - 15$$

$$64.5p^2 + 13p + 6$$

$$65.2x^2 + 11x + 12$$