高三高数复习 3（复数）

1. 解 $z+|z|-5i=8-i$
   Solve $z+|z|-5i=8-i$

$$z=3+4i$$

2. 一个 复数的共轭$z=x+yi$ 是 $\bar{z}=x-yi$. 在一阿尔干图，原点为 O, 点 A 表示 z 和点 B 表示$\frac{1}{\bar{z}}$. 试证明O, A 和 B 共线
   The conjugate of the complex number $z=x+yi$ is $\bar{z}=x-yi$. In an Argand diagram with origin O, the point A represents z and B represent $\frac{1}{\bar{z}}$.Prove that O, A and B are collinear

3. 求一轨迹方程若点 z 移动随 $|z-2+3i|=1$

   Find the equation of the locus of a point z which moves such that $|z-2+3i|=1$

$$x^2+y^2-4x+6y+12=0$$

4. 已知 $z=(\cos\theta+i\sin\theta)$,证明 $\cos 3\theta=4cos^3\theta-3\cos\theta, sin3\theta=3sin\theta-4sin^3\theta$

   Given $z=(\cos\theta+i\sin\theta)$,prove $\cos 3\theta=4cos^3\theta-3\cos\theta, sin3\theta=3sin\theta-4sin^3\theta$

5. 已知二复数为$z = r(\cos\theta_{+} i\sin\theta)$和$w = z + \frac{1}{z}$. 求点 w的轨迹方程式若$r = 2, \theta \in [0,2\pi]$
   Given two complex number $z = r(\cos\theta_{+} i\sin\theta)$ and $w = z + \frac{1}{z}$. Find the equation of the locus of the point w if $r = 2, \theta \in [0,2\pi]$

$$\frac{4}{25}x^2 + \frac{4}{9}y^2 = 1$$

6. 若 $z = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$,求 $z^{65} + z^{66} + z^{67} + \cdots + z^{365}$
   If $z = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$,find $z^{65} + z^{66} + z^{67} + \cdots + z^{365}$

z

7. 若方程式$x^3-9\sqrt{2}x^2+46x-30\sqrt{2}=0$ 的根皆以等差排列，求方程式的三个根。
   If the roots of the equation $x^3-9\sqrt{2}x^2+46x-30\sqrt{2}=0$ are in AP, find the three roots of the equation.

$\sqrt{2},3\sqrt{2},5\sqrt{2}$

8. 已知 α,β,γ 为方程式$x^2+6x^2+3x-2=0$的根，求一方程式若根为 $2-\alpha,2-\beta,2-\gamma$
   Given α,β,γ are roots of the equation $x^2+6x^2+3x-2=0$, form a equation with roots $2-\alpha,2-\beta,2-\gamma$

$x^3-12x^2+39x-36=0$

9. 若 α, β, γ 为方程式$x^3 + 2x^2 + 3x + 4 = 0$的根，求值

   If α, β, γ are the roots of $x^3 + 2x^2 + 3x + 4 = 0$, find the value of

   a. $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$-\frac{3}{4}$

   b. $\alpha^2 + \beta^2 + \gamma^2$

-2

c. $\alpha^3+\beta^3+\gamma^3$

-2

10. 已知 $3i-1$ 为方程式$x^4+x^3+9x^2-8x+10=0$的一根，解方程式

Given $3i-1$ is a root of the equation $x^4+x^3+9x^2-8x+10=0$, solve the equation

$$x=\frac{1+\sqrt{3}i}{2}$$

11. $z=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$, 求 $1+z+z^2+\cdots+z^{12}$

$z=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$, find $1+z+z^2+\cdots+z^{12}$

12. 若 $x^2+x+1=0$, 求 $x^{100}+\frac{1}{x^{10}}$

If $x^2+x+1=0$, find $x^{100}+\frac{1}{x^{10}}$

13. 计算$(\frac{2+2i}{1-\sqrt{3}i})^8$

Calculate $(\frac{2+2i}{1-\sqrt{3}i})^8$

$-8(1-\sqrt{3}i)$

14. 计算 $1 \cdot i \cdot i^2 \cdot i^3 \cdots i^{100}$

Evaluate $1 \cdot i \cdot i^2 \cdot i^3 \cdots i^{100}$

$= -1$