

高三高数复习3 (复数)

1. 解  $z + |z| - 5i = 8 - i$

Solve  $z + |z| - 5i = 8 - i$

$z = 3 + 4i$

2. 一个复数的共轭  $z = x + yi$  是  $\bar{z} = x - yi$ . 在一阿尔干图, 原点为  $O$ , 点  $A$  表示  $z$  和点  $B$  表示  $\frac{1}{z}$ . 试证明  $O, A$  和  $B$  共线

The conjugate of the complex number  $z = x + yi$  is  $\bar{z} = x - yi$ . In an Argand diagram with origin  $O$ , the point  $A$  represents  $z$  and  $B$  represent  $\frac{1}{z}$ . Prove that  $O, A$  and  $B$  are collinear

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3. 求一轨迹方程若点  $z$  移动随  $|z - 2 + 3i| = 1$

Find the equation of the locus of a point  $z$  which moves such that  $|z - 2 + 3i| = 1$

$$x^2 + y^2 - 4x + 6y + 12 = 0$$

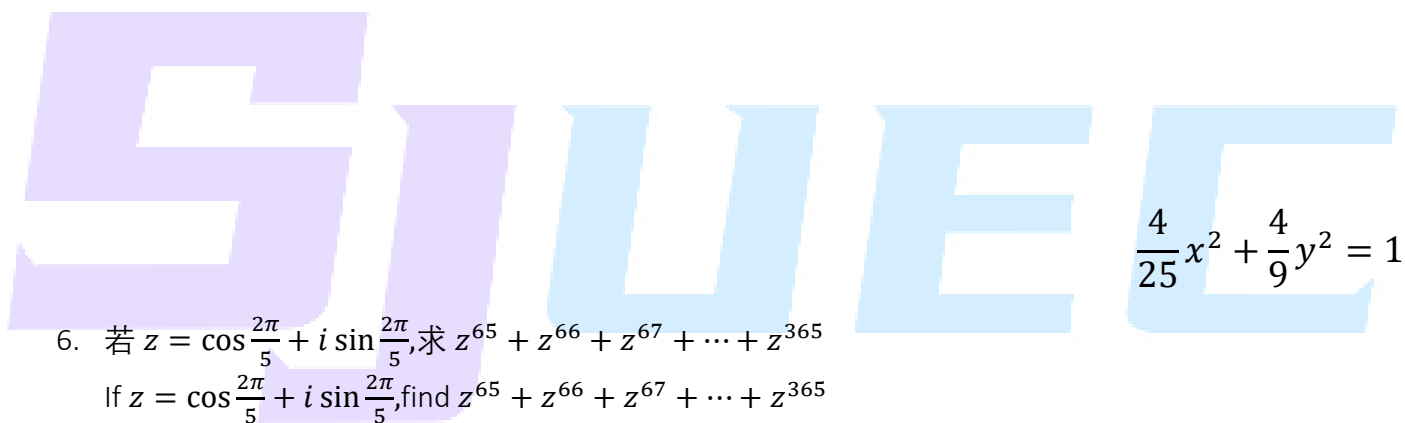
4. 已知  $z = (\cos \theta + i \sin \theta)$ , 证明  $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ ,  $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

Given  $z = (\cos \theta + i \sin \theta)$ , prove  $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ ,  $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

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5. 已知二复数为 $z = r(\cos \theta + i \sin \theta)$ 和 $w = z + \frac{1}{z}$ . 求点  $w$ 的轨迹方程式若 $r = 2, \theta \in [0, 2\pi]$

Given two complex number  $z = r(\cos \theta + i \sin \theta)$  and  $w = z + \frac{1}{z}$ . Find the equation of the locus of the point  $w$  if  $r = 2, \theta \in [0, 2\pi]$



6. 若  $z = \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$ , 求  $z^{65} + z^{66} + z^{67} + \dots + z^{365}$   
If  $z = \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$ , find  $z^{65} + z^{66} + z^{67} + \dots + z^{365}$

$$\frac{4}{25}x^2 + \frac{4}{9}y^2 = 1$$

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7. 若方程式  $x^3 - 9\sqrt{2}x^2 + 46x - 30\sqrt{2} = 0$  的根皆以等差排列, 求方程式的三个根。

If the roots of the equation  $x^3 - 9\sqrt{2}x^2 + 46x - 30\sqrt{2} = 0$  are in AP, find the three roots of the equation.



8. 已知  $\alpha, \beta, \gamma$  为方程式  $x^2 + 6x^2 + 3x - 2 = 0$  的根, 求一方程式若根为  $2 - \alpha, 2 - \beta, 2 - \gamma$

Given  $\alpha, \beta, \gamma$  are roots of the equation  $x^2 + 6x^2 + 3x - 2 = 0$ , form a equation with roots  $2 - \alpha, 2 - \beta, 2 - \gamma$

$$x^3 - 12x^2 + 39x - 36 = 0$$

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9. 若  $\alpha, \beta, \gamma$  为方程式  $x^3 + 2x^2 + 3x + 4 = 0$  的根, 求值

If  $\alpha, \beta, \gamma$  are the roots of  $x^3 + 2x^2 + 3x + 4 = 0$ , find the value of

a.  $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

b.  $\alpha^2 + \beta^2 + \gamma^2$



c.  $a^3 + \beta^3 + \gamma^3$

10. 已知  $3i - 1$  为方程式  $x^4 + x^3 + 9x^2 - 8x + 10 = 0$  的一根, 解方程式

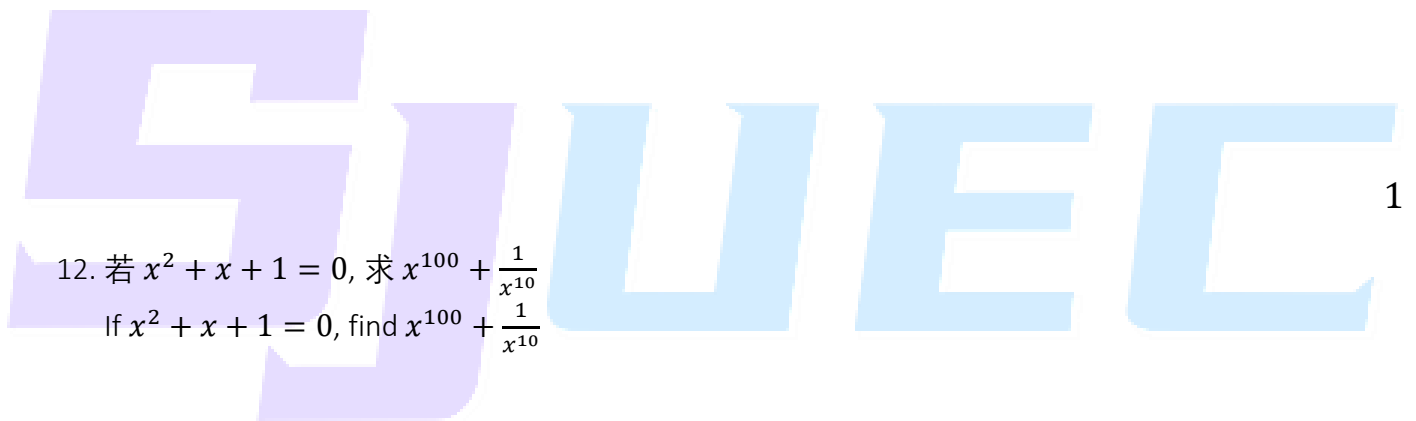
Given  $3i - 1$  is a root of the equation  $x^4 + x^3 + 9x^2 - 8x + 10 = 0$ , solve the equation

$$x = \frac{1 + \sqrt{3}i}{2}$$

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11.  $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ , 求  $1 + z + z^2 + \dots + z^{12}$

$z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ , find  $1 + z + z^2 + \dots + z^{12}$



12. 若  $x^2 + x + 1 = 0$ , 求  $x^{100} + \frac{1}{x^{10}}$   
If  $x^2 + x + 1 = 0$ , find  $x^{100} + \frac{1}{x^{10}}$

= -1

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13. 计算  $(\frac{2+2i}{1-\sqrt{3}i})^8$

Calculate  $(\frac{2+2i}{1-\sqrt{3}i})^8$



14. 计算  $1 \cdot i \cdot i^2 \cdot i^3 \dots i^{100}$   
Evaluate  $1 \cdot i \cdot i^2 \cdot i^3 \dots i^{100}$

$-8(1 - \sqrt{3}i)$

$= -1$