1. 利用数学归纳法，试证明$2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + (n+1) \cdot 2^n = n \cdot 2^{n+1}$

   Using mathematical induction, prove that $2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + (n+1) \cdot 2^n = n \cdot 2^{n+1}$

2. 利用数学归纳法证明 $3^{4n+2} + 5^{2n+1}$ 可被 14 整除，且 $n \epsilon N$

   Prove by mathematical induction that $3^{4n+2} + 5^{2n+1}$ is divisible by 14 for $n \epsilon N$

3. 已知 $n \epsilon N, n > 4$,试以数学归纳法证明$2^n > n^2$

   Given that $n \epsilon N, n > 4$,prove by the principle of mathematical induction that $2^n > n^2$

4. 以数学归纳法证明$1 \times 1! + 2 \times 2! + 3 \times 3! + \cdots + n \times n! = (n+1)! - 1$

   Prove by mathematical induction $1 \times 1! + 2 \times 2! + 3 \times 3! + \cdots + n \times n! = (n+1)! - 1$

5. 以数学归纳法证明$\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)\ldots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$ $(n>1)$. 据此，计算 $\left(1-\frac{1}{441}\right)\left(1-\frac{1}{484}\right)\ldots\left(1-\frac{1}{900}\right)$

   Prove by the principle of mathematical induction that $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)\ldots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$ $(n>1)$. Hence, evaluate $\left(1-\frac{1}{441}\right)\left(1-\frac{1}{484}\right)\ldots\left(1-\frac{1}{900}\right)$

$\frac{62}{63}$

6. 以数学归纳法证明 $\sum_{r=1}^{n}\frac{r}{(r+1)!}=1-\frac{1}{(n+1)!}$

   Prove by mathematical induction$\sum_{r=1}^{n}\frac{r}{(r+1)!}=1-\frac{1}{(n+1)!}$

7. 不用计算机，求$\sin(sin^{-1}\frac{1}{\sqrt{5}}+tan^{-1}\frac{1}{3})$

Without using calculator, find $\sin(sin^{-1}\frac{1}{\sqrt{5}}+tan^{-1}\frac{1}{3})$

$\frac{\sqrt{2}}{2}$

8. 解 $sin^{-1}x = cos^{-1}x$, 若 $0 \le x \le 1$

Solve $sin^{-1}x = cos^{-1}x$, for $0 \le x \le 1$

$\frac{\sqrt{2}}{2}$

9. 解 $tan^{-1}x + tan^{-1}\frac{2(1-x)}{3} = \frac{\pi}{4}$

Solve $tan^{-1}x + tan^{-1}\frac{2(1-x)}{3} = \frac{\pi}{4}$

$$x = 1, x = \frac{1}{2}$$

10. 解 $sin^{-1}x + sin^{-1}(\sqrt{3}x) = \frac{\pi}{2}$

Solve $sin^{-1}x + sin^{-1}(\sqrt{3}x) = \frac{\pi}{2}$

$$x = \frac{1}{2}$$

11. 解 $tan^{-1}x$=$cot^{-1}x$

Solve $tan^{-1}x$=$cot^{-1}x$

$x = 1$

12. 解 $cos^{-1}x - sin^{-1}x = cos^{-1}\frac{\sqrt{3}}{2}$

Solve $cos^{-1}x - sin^{-1}x = cos^{-1}\frac{\sqrt{3}}{2}$

$x = \frac{1}{2}$

13. 已知 $a \geq 0, b \geq 0$ 和 $(a+1)(b+1)=2$, 求 $tan^{-1}a + tan^{-1}b$

Given that $a \geq 0, b \geq 0$ and $(a+1)(b+1)=2$, find $tan^{-1}a + tan^{-1}b$

$\frac{\pi}{4}$

14. 解 $cos^{-1}x - sin^{-1}x = cos^{-1}(x\sqrt{3})$ 若角度皆是锐角

Solve $cos^{-1}x - sin^{-1}x = cos^{-1}(x\sqrt{3})$ where all angles are acute

$x = \frac{1}{2}$

15. 已知$y = sin^{-1}\frac{x}{3}$, 求 $\frac{dy}{dx}$

Given $y = sin^{-1}\frac{x}{3}$, find $\frac{dy}{dx}$

$$\frac{1}{\sqrt{9-x^2}}$$

16. $y = tan^{-1}\frac{x}{1-x^2}$, 求 $\frac{dy}{dx}$

$y = tan^{-1}\frac{x}{1-x^2}$, find $\frac{dy}{dx}$

$$\frac{1+x^2}{x^4-x^2+1}$$

17. 若$y = (sin^{-1}x)^2$, 试证明 $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$

$y = (sin^{-1}x)^2$, prove $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$

18. $y = cos^{-1}\left(2x\sqrt{1 - x^2}\right)$, 试证明 $\frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$

$y = cos^{-1}\left(2x\sqrt{1 - x^2}\right)$, show that $\frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$

19. 求直线方程式若一斜率为-1 的直线是曲线 $xy+3x-2y-10=0$的切线

Find the equation(s) of the tangent(s) of gradient -1 to the curve $xy+3x-2y-10=0$

$x+y-3=0, x+y+5=0$

20. $y=(4+x^2)tan^{-1}\frac{x}{2}$, 求 $\frac{dy}{dx}$

$y=(4+x^2)tan^{-1}\frac{x}{2}$, find $\frac{dy}{dx}$

$2+2xtan^{-1}\frac{x}{2}$