

高三高数练习 2 (数学归纳法、反三角函数、微分法)

1. 利用数学归纳法, 试证明 $2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + (n+1) \cdot 2^n = n \cdot 2^{n+1}$

Using mathematical induction, prove that $2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + (n+1) \cdot 2^n = n \cdot 2^{n+1}$



2. 利用数学归纳法证明 $3^{4n+2} + 5^{2n+1}$ 可被 14 整除, 且 $n \in N$

Prove by mathematical induction that $3^{4n+2} + 5^{2n+1}$ is divisible by 14 for $n \in N$

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3. 已知 $n \in N, n > 4$, 试以数学归纳法证明 $2^n > n^2$

Given that $n \in N, n > 4$, prove by the principle of mathematical induction that $2^n > n^2$



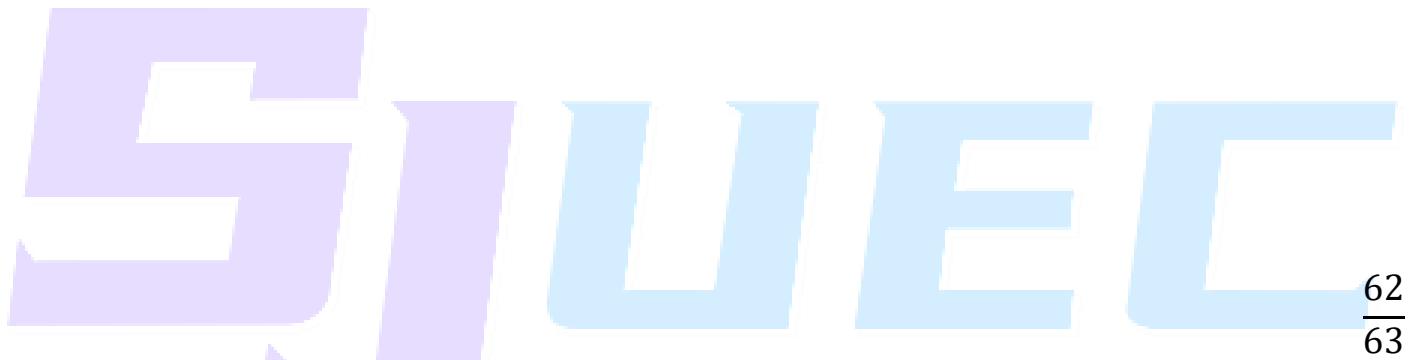
4. 以数学归纳法证明 $1 \times 1! + 2 \times 2! + 3 \times 3! + \cdots + n \times n! = (n + 1)! - 1$

Prove by mathematical induction $1 \times 1! + 2 \times 2! + 3 \times 3! + \cdots + n \times n! = (n + 1)! - 1$

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5. 以数学归纳法证明 $\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\left(1 - \frac{1}{16}\right) \dots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$ ($n > 1$). 据此, 计算 $\left(1 - \frac{1}{441}\right)\left(1 - \frac{1}{484}\right) \dots \left(1 - \frac{1}{900}\right)$

Prove by the principle of mathematical induction that $\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\left(1 - \frac{1}{16}\right) \dots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$ ($n > 1$). Hence, evaluate $\left(1 - \frac{1}{441}\right)\left(1 - \frac{1}{484}\right) \dots \left(1 - \frac{1}{900}\right)$



$\frac{62}{63}$

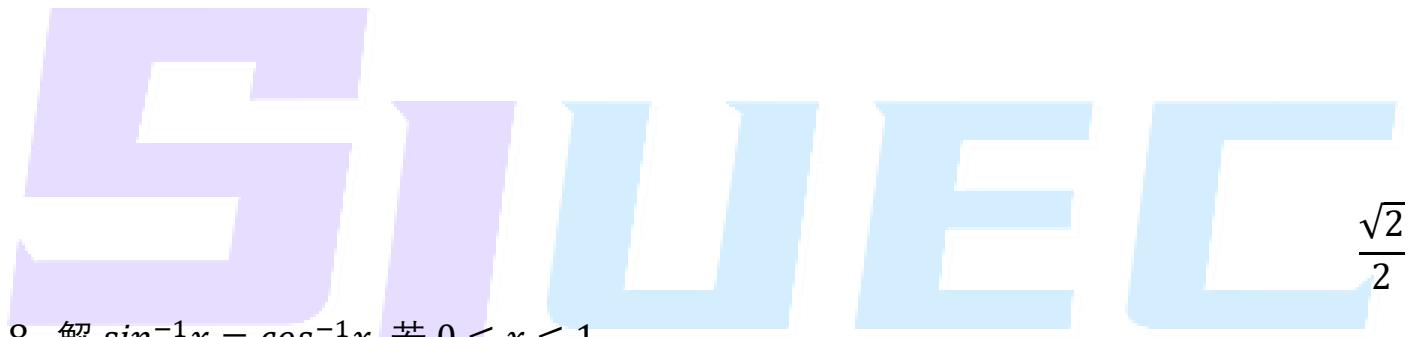
6. 以数学归纳法证明 $\sum_{r=1}^n \frac{r}{(r+1)!} = 1 - \frac{1}{(n+1)!}$

Prove by mathematical induction $\sum_{r=1}^n \frac{r}{(r+1)!} = 1 - \frac{1}{(n+1)!}$

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7. 不用计算机, 求 $\sin(\sin^{-1}\frac{1}{\sqrt{5}} + \tan^{-1}\frac{1}{3})$

Without using calculator, find $\sin(\sin^{-1}\frac{1}{\sqrt{5}} + \tan^{-1}\frac{1}{3})$



8. 解 $\sin^{-1}x = \cos^{-1}x$, 若 $0 \leq x \leq 1$

Solve $\sin^{-1}x = \cos^{-1}x$, for $0 \leq x \leq 1$

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9. 解 $\tan^{-1}x + \tan^{-1}\frac{2(1-x)}{3} = \frac{\pi}{4}$

Solve $\tan^{-1}x + \tan^{-1}\frac{2(1-x)}{3} = \frac{\pi}{4}$

10. 解 $\sin^{-1}x + \sin^{-1}(\sqrt{3}x) = \frac{\pi}{2}$

Solve $\sin^{-1}x + \sin^{-1}(\sqrt{3}x) = \frac{\pi}{2}$

$x = 1, x = \frac{1}{2}$

$x = \frac{1}{2}$

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11. 解 $\tan^{-1}x = \cot^{-1}x$

Solve $\tan^{-1}x = \cot^{-1}x$

12. 解 $\cos^{-1}x - \sin^{-1}x = \cos^{-1}\frac{\sqrt{3}}{2}$

Solve $\cos^{-1}x - \sin^{-1}x = \cos^{-1}\frac{\sqrt{3}}{2}$

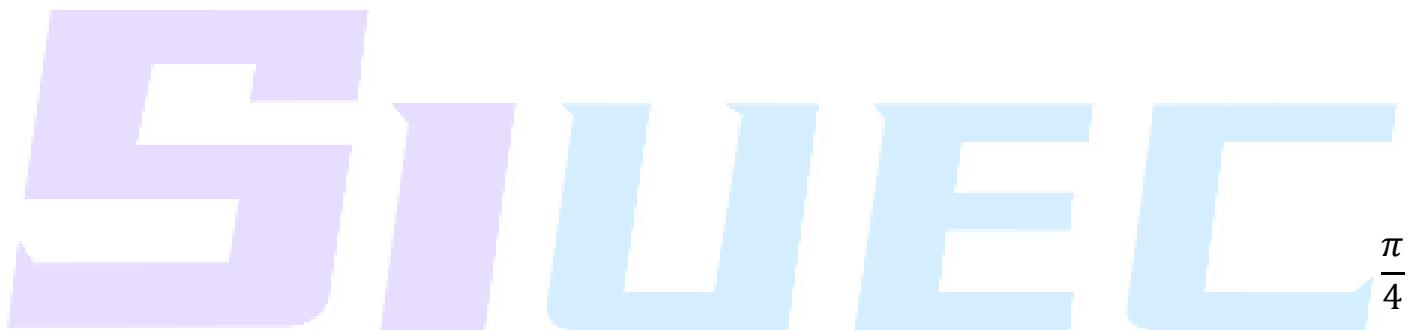
$x = 1$

$x = \frac{1}{2}$

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13. 已知 $a \geq 0, b \geq 0$ 和 $(a + 1)(b + 1) = 2$, 求 $\tan^{-1}a + \tan^{-1}b$

Given that $a \geq 0, b \geq 0$ and $(a + 1)(b + 1) = 2$, find $\tan^{-1}a + \tan^{-1}b$



14. 解 $\cos^{-1}x - \sin^{-1}x = \cos^{-1}(x\sqrt{3})$ 若角度皆是锐角

Solve $\cos^{-1}x - \sin^{-1}x = \cos^{-1}(x\sqrt{3})$ where all angles are acute

$$x = \frac{1}{2}$$

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15. 已知 $y = \sin^{-1} \frac{x}{3}$, 求 $\frac{dy}{dx}$

Given $y = \sin^{-1} \frac{x}{3}$, find $\frac{dy}{dx}$

16. $y = \tan^{-1} \frac{x}{\sqrt{9-x^2}}$, 求 $\frac{dy}{dx}$

$y = \tan^{-1} \frac{x}{\sqrt{9-x^2}}$, find $\frac{dy}{dx}$

$$\frac{1}{\sqrt{9-x^2}}$$

$$\frac{1+x^2}{x^4-x^2+1}$$

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17. 若 $y = (\sin^{-1}x)^2$, 试证明 $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 2$

$$y = (\sin^{-1}x)^2, \text{ prove } (1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 2$$

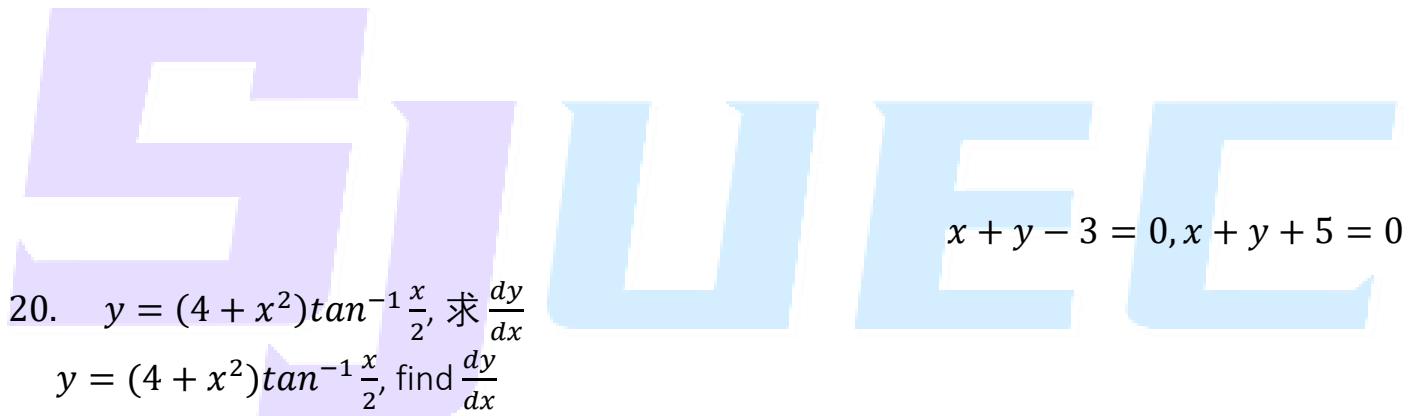
18. $y = \cos^{-1}(2x\sqrt{1-x^2})$, 试证明 $\frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$

$$y = \cos^{-1}(2x\sqrt{1-x^2}), \text{ show that } \frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$$

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19. 求直线方程式若一斜率为 -1 的直线是曲线 $xy + 3x - 2y - 10 = 0$ 的切线

Find the equation(s) of the tangent(s) of gradient -1 to the curve $xy + 3x - 2y - 10 = 0$



20. $y = (4 + x^2)\tan^{-1}\frac{x}{2}$, 求 $\frac{dy}{dx}$

$y = (4 + x^2)\tan^{-1}\frac{x}{2}$, find $\frac{dy}{dx}$

$$2 + 2x\tan^{-1}\frac{x}{2}$$