高三高数复习 1 （行列式、矩阵）

1 试证明$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

Prove $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

2. 试证明$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$

Prove $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$

3. 试证明 $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

Prove $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

4. 解 $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$

Solve $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$

$x = -\frac{a}{3}$

5. $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0.$ 试证明 $a+b+c=0, a=b=c$

$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0.$ Prove $a+b+c=0, a=b=c$

6. 计算 $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

Evaluate $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

7. 试证明 $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

Prove $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

8. 试证明 $\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

Prove $\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

9. 求矩阵 $A=\begin{pmatrix}1 & -1 & 1\\ 4 & -4 & 1\\ 1 & 2 & -1\end{pmatrix}$的反矩阵.据此，求解联立方程式$\begin{cases}x-y+z=4\\ 4x-4y+z=7\\ x+2y-z=1\end{cases}$

Find the inverse matric of $A=\begin{pmatrix}1 & -1 & 1\\ 4 & -4 & 1\\ 1 & 2 & -1\end{pmatrix}$.Hence or otherwise, solve the simultaneous linear equation $\begin{cases}x-y+z=4\\ 4x-4y+z=7\\ x+2y-z=1\end{cases}$

$\begin{pmatrix}\frac{2}{9} & \frac{1}{9} & \frac{1}{3}\\ \frac{5}{9} & -\frac{2}{9} & \frac{1}{3}\\ \frac{4}{3} & -\frac{1}{3} & 0\end{pmatrix}; x=2, y=1, z=3$

10. 证明$\begin{vmatrix}1 & a & bc\\ 1 & b & ca\\ 1 & c & ab\end{vmatrix}=(a-b)(b-c)(c-a)$.据此，推断$\begin{vmatrix}1 & b & ca\\ 1 & a & bc\\ 3 & 3c & 3ab\end{vmatrix}$.

Prove that $\begin{vmatrix}1 & a & bc\\ 1 & b & ca\\ 1 & c & ab\end{vmatrix}=(a-b)(b-c)(c-a)$.Hence, deduce $\begin{vmatrix}1 & b & ca\\ 1 & a & bc\\ 3 & 3c & 3ab\end{vmatrix}$.

$-3(a-b)(b-c)(c-a)$

11 求矩阵$A=\begin{pmatrix}3 & -1 & 2\\ 2 & 1 & -4\\ 1 & 1 & 3\end{pmatrix}$的逆矩阵。据此，求解联立方程式$\begin{cases}3x-y+2z=0\\ 2x+y-4z=-9\\ x+y+3z=6\end{cases}$.

Find the inverse matric of $A=\begin{pmatrix}3 & -1 & 2\\ 2 & 1 & -4\\ 1 & 1 & 3\end{pmatrix}$.Hence or otherwise, solve the simultaneous linear equation $\begin{cases}3x-y+2z=0\\ 2x+y-4z=-9\\ x+y+3z=6\end{cases}$.

$$\begin{pmatrix}\frac{7}{33} & \frac{5}{33} & \frac{2}{33}\\ -\frac{10}{33} & \frac{7}{33} & \frac{16}{33}\\ \frac{1}{33} & -\frac{4}{33} & \frac{5}{33}\end{pmatrix};x=-1,y=1,z=2$$

12. 用行列式的属性，试证明$\begin{vmatrix}a & 1 & a^2(b+c)\\ b & 1 & b^2(c+a)\\ c & 1 & c^2(a+b)\end{vmatrix}=0$

Using the properties of determinant, prove that $\begin{vmatrix}a & 1 & a^2(b+c)\\ b & 1 & b^2(c+a)\\ c & 1 & c^2(a+b)\end{vmatrix}=0$

13. 解方程式$\begin{vmatrix} x & 1 & x+1 \\ 2 & x & 3 \\ 1 & 3 & -1 \end{vmatrix} = -2x^2 + 11$

solve the equation $\begin{vmatrix} x & 1 & x+1 \\ 2 & x & 3 \\ 1 & 3 & -1 \end{vmatrix} = -2x^2 + 11$

0

14. 已知$A = \begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}, B = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$和 $AX = B$,

a) 求矩阵 A 的反矩阵

b) 据此，求矩阵 X

Given that $A = \begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}, B = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$ and $AX = B$,

a) find the inverse matrix of A

b) Hence, find the matrix X

a) $A^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ -38 & 41 & -34 \\ 27 & -29 & 24 \end{pmatrix}$

b) $x = \begin{pmatrix} 0 & 0 & -2 \\ 4 & 3 & 75 \\ -3 & -2 & -53 \end{pmatrix}$

15. 利用克兰姆定律解方程式$\begin{cases} x+2y+3z=14 \\ 3x+2y+z=10 \\ 3x+y+2z=11 \end{cases}$

Use the Cramer's Rule to solve the simultaneous equation $\begin{cases} x+2y+3z=14 \\ 3x+2y+z=10 \\ 3x+y+2z=11 \end{cases}$

$x=1, y=2, z=3$