

Trigonometry Revision 3

1. By considering $\sin(45^\circ + 30^\circ)$, prove that $\sin 75^\circ = \frac{1+\sqrt{3}}{2\sqrt{2}}$
2. Prove that $2 \cos(\theta - \frac{\pi}{3}) = \cos \theta + \sqrt{3} \sin \theta$
3. Solve the equation $\cos(\theta + 60^\circ) = 2 \sin(\theta - 45^\circ)$ for $0^\circ \leq \theta \leq 360^\circ$
4. Prove that $\tan(\theta + 45^\circ) = \frac{1+\tan \theta}{1-\tan \theta}$
5. Show that $\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1}$
6. Given that $\sin \alpha = \frac{4}{5}$ and $\sin \beta = \frac{5}{13}$ where α and β are acute angles, find the exact values of
 - a. $\cos \alpha$ and $\cos \beta$
 - b. $\sin(\alpha + \beta)$, $\sin(\alpha - \beta)$, $\cos(\alpha + \beta)$, $\cos(\alpha - \beta)$
 - c. $\tan(\alpha + \beta)$, $\tan(\alpha - \beta)$
7. Find, the exact form, the values of
 - a. $\cos 10^\circ \cos 20^\circ - \sin 10^\circ \sin 20^\circ$
 - b. $\sin 75^\circ \cos 45^\circ + \cos 75^\circ \sin 45^\circ$
 - c. $\frac{\tan 103^\circ - \tan 58^\circ}{1 + \tan 103^\circ \tan 58^\circ}$
 - d. $\frac{\tan 75^\circ + 1}{\tan 75^\circ - 1}$
8. Simplify
 - a. $\sin(\alpha + \beta) + \sin(\alpha - \beta)$
 - b. $\cos(\alpha + \beta) + \cos(\alpha - \beta)$
 - c. $\sin(\alpha - \beta) \cos \alpha - \cos(\alpha - \beta) \sin \alpha$
9. Use the addition formulae to show that
 - a. $\sin\left(\theta + \frac{\pi}{2}\right) = \cos \theta$
 - b. $\sin(90^\circ - \theta) = \cos \theta$
 - c. $\cos(90^\circ - \theta) = \sin \theta$
 - d. $\cos(180^\circ + \theta) = -\cos \theta$
 - e. $\tan(\theta + 180^\circ) = \tan \theta$

- f. $\tan(\pi - \theta) = -\tan \theta$
10. Prove the identity $\operatorname{cosec}(\theta + \varphi) = \frac{\operatorname{cosec} \theta \operatorname{cosec} \varphi}{\cot \theta + \cot \varphi}$
11. Express $\cot(\theta + \varphi)$ in terms of $\cot \theta$ and $\cot \varphi$
12. Prove that $\tan(\alpha + \beta + \gamma) = \frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \tan \beta \tan \gamma}{1 - \tan \alpha \tan \beta - \tan \beta \tan \gamma - \tan \alpha \tan \gamma}$
13. Solve the equation $\tan \theta = 2 \tan(45^\circ - \theta)$ for $0^\circ \leq \theta \leq 360^\circ$
14. Solve the equation $\cos\left(\theta - \frac{\pi}{6}\right) = \cos\left(\theta + \frac{\pi}{6}\right)$ for $0 \leq \theta \leq 2\pi$
15. Solve the equation $5 \cos(\theta + 45^\circ) = \sin(\theta - 45^\circ)$ for $0^\circ \leq \theta \leq 360^\circ$
16. Solve the following equations for $0^\circ \leq \theta \leq 360^\circ$
- $4 \sin 2\theta = \sin \theta$
 - $\cos 2\theta = \cos \theta$
17. Express $\cos 3\theta$ in terms of $\cos \theta$
18. Solve these equation for $0^\circ \leq \theta \leq 360^\circ$
- $\sin 2\theta = \cos \theta$
 - $\sin 2\theta - \sqrt{3} \cos \theta = 0$
 - $\cos 2\theta + 3 \cos \theta - 1 = 0$
 - $4 \cos 2\theta + 2 \sin \theta - 1 = 0$
 - $\tan 2\theta + \tan \theta = 0$
 - $\sin 2\theta = \tan \theta$
19. Solve these equations for $0 \leq \theta \leq 2\pi$
- $2 \tan 2\theta = 5 \tan \theta$
 - $5 \sin 2\theta = 2 \sin \theta$
 - $2 \cos 2\theta = 1 - 4 \cos \theta$
20. An acute angle A is such that $\sin \theta = \frac{3}{5}$, find the exact value of
- $\cos 2\theta$
 - $\tan 2\theta$
 - $\tan 4\theta$
21. If $\tan \theta = 3$, where $180^\circ \leq \theta \leq 270^\circ$, find the exact values of
- $\sin \theta$

- b. $\cos \theta$
- c. $\tan 2\theta$
- d. $\cos 2\theta$
22. Express $(\sin x - \cos x)^2$ in terms of $\sin 2x$
23. Express $(\cos^4 x - \sin^4 x)(\cos^2 x - \sin^2 x)$ in terms of $\cos 2x$
24. Express $\sin 4\theta \sin \theta$ in terms of $\cos \theta$
25. a. i. By considering $\sin(2\theta + \theta)$, express $\sin 3\theta$ in terms of $\sin \theta$
 ii. Solve the equation $\sin 3\theta = \sin \theta$ for $0^\circ \leq \theta \leq 360^\circ$
- b. Express $\tan 3\theta$ in terms of $\tan \theta$
26. Express $\cot 2\theta$ in terms of $\cot \theta$
27. Prove that
- a. $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$
- b. $\frac{\sin 2\theta}{1 - \cos 2\theta} = \cot \theta$
28. Prove that $\frac{\sin \alpha}{\cos \beta} + \frac{\cos \alpha}{\sin \beta} = \frac{2 \cos(\alpha - \beta)}{\sin 2\beta}$
29. Prove that $\sec 2\theta - \tan 2\theta = \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}$

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