Trigonometry revision 4

1. a) Express $2\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° \leq \alpha \leq 90°$, giving the exact value of R and the value of α correct to 2 decimal places.

b) Hence, solve the equation $2\sin\theta + \cos\theta = 2$ for $0° \leq \theta \leq 360°$

2. Express $3\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0° \leq \alpha \leq 90°$, giving the exact value of R and the value of α.

b) Find the greatest and least possible values of $[(3\cos\theta + \sqrt{3}\sin\theta)^2 - 5]$ as θ varies

3. a)Express each of the following in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0° \leq \alpha \leq 90°$, giving the exact value of R and the value of α to correct to 2 decimal places

i. $3\cos\theta + 4\sin\theta$

ii. $5\cos\theta + 12\sin\theta$

iii. $\sqrt{2}\cos\theta + \sin\theta$

iv. $\sin\theta + \cos\theta$

b) Find the greatest and least possible values of each of the expressions in part (a)

c) For each of the expressions in part (a) find a value of θ for which the expression has its greatest value

5.a) Express $\sin\theta + 2\cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° \leq \alpha \leq 90°$, giving the exact value of R and the value of α correct to 2 decimal places.

b) Hence, solve the equation $\sin\theta + 2\cos\theta = 1$ for $0° \leq \theta \leq 360°$

6. a) Express $\sqrt{3}\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$ where $R > 0$ and $0° \leq \alpha \leq 90°$, giving the exact value of R and α.

b) Hence, solve the equation $\sqrt{3}\sin\theta - \cos\theta = 1$ for $0° \leq \theta \leq 360°$

c) Find the greatest and least possible values of $(\sqrt{3}\sin\theta - \cos\theta)^2$as θ varies

7.a) Express $2\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$ where $R > 0$ and $0° \leq \alpha \leq 90°$, giving the exact value of R and α.

b) Hence, solve the equation $\cos\theta - \sin\theta = \frac{1}{2}$ for $0° \leq \theta \leq 360°$

8. a) Express $4\cos\theta + 6\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0° \leq \alpha \leq 90°$, giving the exact value of R and the value of α correct to 2 decimal places.

b) Hence, solve the equation $4\cos\theta + 6\sin\theta = 5$ for $0° \leq \theta \leq 360°$

c) Find the greatest and least possible values of $[(4\cos\theta + 6\sin\theta)^2 + 5]$ as θ varies

9. a) Express $2\sin 2x + \cos 2x$ in the form $R\sin(2x + \alpha)$ where $R > 0$ and $0° \leq \alpha \leq 90°$, giving the exact value of R and the value of α correct to 2 decimal places.

b) Hence, solve the equation $2\sin 2x + \cos 2x = 1$for $0° \leq x \leq 360°$

c) Find the greatest and least possible values of $10 - (2\sin 2x + \cos 2x)$ as x varies

10. a) Express $\sin x + 4\cos x$ in the form $R\sin(x + \alpha)$ where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$, giving the exact value of R and the value of α correct to 3 decimal places.

b) Hence, solve the equation $\sin x + 4\cos x = 3$for $0 \leq x \leq 2\pi$

c) Find the greatest and least possible values of $[(\sin x + 4\cos x)^2 - 1]$as x varies