

Factoring Rules

$$1. x^2 - (r + s)x + rs = (x - r)(x - s)$$

$$2. x^2 + 2ax + a^2 = (x + a)^2 \text{ and } x^2 - 2ax + a^2 = (x - a)^2$$

$$3. \text{ Difference of squares: } a^2 - b^2 = (a - b)(a + b)$$

$$4. \text{ Difference of cubes: } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$5. a^4 - b^4 = (a - b)(a^3 + a^2b + ab^2 + b^3) = (a - b) [a^2(a + b) + b^2(a + b)] = (a - b)(a + b)(a^2 + b^2)$$

$$\text{or, more simply, } a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a - b)(a + b)(a^2 + b^2)$$

$$6. a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$$

$$7. a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + ab^{n-2} + b^{n-1})$$

$$8. \text{ Sum of cubes: } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$9. a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$$

$$10. a^7 + b^7 = (a + b)(a^6 - a^5b + a^4b^2 - a^3b^3 + a^2b^4 - ab^5 + b^6)$$

$$11. \text{ If } n \text{ is odd, then } a^n + b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \dots + a^2b^{n-3} - ab^{n-2} + b^{n-1})$$

$$12. \text{ Sum of squares: } a^2 + b^2 = (a - bi)(a + bi) \quad \text{Note: } a^2 + b^2 \text{ does not factor using real numbers.}$$

$$13. a^4 + b^4 = (a^2 - \sqrt{2}ab + b^2)(a^2 + \sqrt{2}ab + b^2)$$