

Differentiation 3

1. Find the coordinates of the stationary points of the curve $y = t^{\frac{1}{2}}(\frac{3}{4} - t)$ and determine their nature

2. Find the coordinates of the stationary points of the curve $y = (3x - 2)^3 - 9x$ and determine their nature

3. Find the coordinates of the stationary points of the curve $y = \frac{1+54x^3}{x^2}$ and determine their nature

4. Find the coordinates of the stationary points of the curve $y = x^4 - 2x^3 + x^2 - 2$ and determine their nature. Sketch the curve

5. Show that the curve $y = 3x^3 - 5x^2 + 3x + 4$ has no stationary points

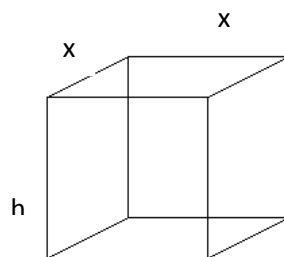
6. $f(x) = 3x^2 + 2x + 5$, $g(x) = x^3 - 4x^2 - 3x + 6$

- Show that there is one value of x for which $f(x)$ and $g(x)$ have the same stationary value
- On the same axes sketch the graph of $f(x)$ and $g(x)$

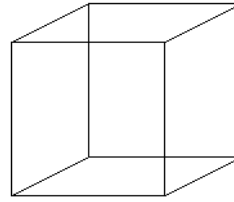
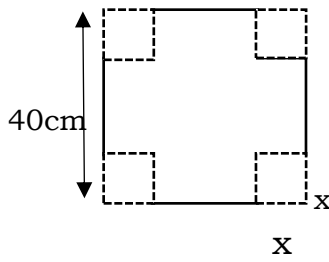
7. The curve $y = ax^2 + bx + c$ has a minimum when $x = \frac{1}{2}$ and passes through the points $(2,0)$ and $(1,-3)$. Find the values of a , b and c

8. A farmer has a rectangular piece of land for pigs. One of the sides of the rectangle is a wall. The other three sides have fencing. The fencing is 80m in length. Find the maximum possible area of this rectangular piece of land

9. An open box with a square base has a total surface of 300cm^2 . Find the greatest possible volume of the box

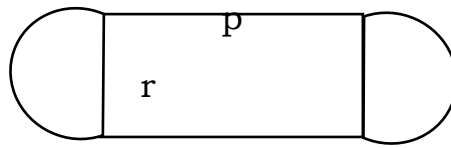


10. Figure one shows a square sheet of metal of side 40cm. A square x cm by x cm is cut from each corner. The sides are then bent upwards to form an open box as shown in figure 2. Find the value of x that maximizes the volume of the box



11. Given that $x + y = 3$, find the least possible value of $x^2 + 14y$

12. The diagram shows a sporting track made up of a rectangle with semicircles at each end. The rectangle has dimensions p metres by $2r$ metres where r is the radius of each semicircle. The perimeter of the track is 1400m. The track has a maximum area for this perimeter. Find the value of p and the value of r .



13. Find the maximum possible value of x^2y if $x + 2y = 8$

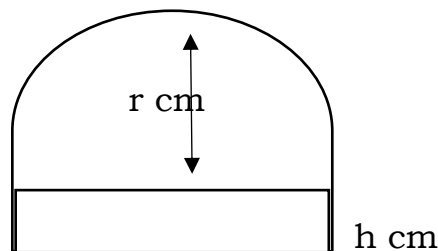
14. A cylinder with an open top has radius r cm and a volume of 512π cm³.

- a. Write down the surface area of the cylinder in terms of r
- b. Find the minimum surface area. Leave your answer in terms of π
- c. Find the value of r and the height of the cylinder

15. A sector of a circle, radius r has a perimeter of 20cm. The angle of the sector is θ radians and the area is A cm². Find the maximum possible area of the sector.

16. A tank in the shape of a right circular cylinder with no top has a surface area of 3π m². What height and base radius will maximize the volume of the cylinder?

17. The diagram shows a semicircle on top of a rectangle. The perimeter of the shape is 20cm. Find the maximum area of the rectangle.



18. $w = pq$ and $2p + 5q = 100$. Find the maximum value of w .

19. A spherical balloon is being blown up so that its radius increases at a rate of 0.4 cm s⁻¹. Find the rate of increase of the surface area of the balloon when the radius is 20cm.

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20. The radius of a circular ink blob is increasing at a rate of 5cm s^{-1} . Find the exact rate of increase of the circumference of the circle.
21. The side of a cube is increasing at 0.2 ms^{-1} . Find the rate of increase of the volume when the length of the side is 4cm .
22. A spherical balloon is inflated so that its volume increases at the rate of $50\text{cm}^3\text{s}^{-1}$. Find the rate of increase of the radius of the balloon when the radius is 12cm .
23. The side of a cube is decreasing at a rate of 0.4cm s^{-1} . Find the rate of decrease of the surface area when the length of the side is 3cm .
24. A cone has a height of 7cm . The radius of the base of the cone is increasing at a rate of 8cm s^{-1} . Find the rate of change of the volume of the cone when the base radius is 5cm .
25. The volume of a cube is increasing at the rate of $12\text{ cm}^3\text{s}^{-1}$. Find the rate of increase of the surface area of the cube when the side of the cube is 7cm .
26. The surface area of a cube is increasing at $0.3\text{m}^2\text{s}^{-1}$. Find the rate of increase of the volume of the cube when the length of the side is 5m .
27. A cuboid has a square base. The height of the cuboid is twice the length of the side of the base. The surface area of the cuboid is increasing at a rate of $10\text{cm}^2\text{s}^{-1}$. Find the rate of increase of the volume of the cuboid when the height of the cuboid is 12cm .

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