Mathematical Induction

1. Using mathematical induction, prove that $2.2 + 3.2^2 + 4.2^2 + \cdots + (n+1).2^n = n.2^{n+1}$
2. Prove by mathematical induction that $3^{4n+2} + 5^{2n+1}$ is divisible by 14 for $n \in N$
3. Given that $n \epsilon N, n > 4$,prove by the principle of mathematical induction that $2^n > n^2$
4. Prove by mathematical induction $1 \times 1! + 2 \times 2! + 3 \times 3! + \cdots + n \times n! = (n+1)! - 1$
5. Prove by the principle of mathematical induction that $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)\ldots\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}$ $(n > 1)$. Hence, evaluate $\left(1-\frac{1}{441}\right)\left(1-\frac{1}{484}\right)\ldots\left(1-\frac{1}{900}\right)$
6. Prove by mathematical induction $\sum_{r=1}^{n} \frac{r}{(r+1)!} = 1 - \frac{1}{(n+1)!}$
7. Prove using mathematical induction that for all $n \geq 1, 1 + 4 + 7 + \cdots + (3n-2) = \frac{n(3n-1)}{2}$
8. Using the Principle of Mathematical Induction to verify that, for n any positive integer, $6^n - 1$ is divisible by 5
9. Consider the sequence of real numbers defined by the relations

   $x_1 = 1$ and $x_{n+1=}\sqrt{1+2x_n}$ for $n \geq 1$. Use the Principle of Mathematical Induction to show that $x_n < 4$ for all

   $n \geq 1$
10. Show that $n! > 3^n$ for $n \geq 7$
11. Using the principle of mathematical induction, prove that $1^2 + 2^2 + 3^2 + \ldots.. + n^2 = \frac{1}{6}[n(n + 1)(2n + 1)]$ for all n ∈ N
12. By using mathematical induction prove that the given equation is true for all positive integers.

    1 x 2 + 3 x 4 + 5 x 6 + ... + (2n - 1) x 2n = $\frac{n(n+1)(4n-1)}{3}$
13. Using the principle of mathematical induction, prove that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots.. + n(n + 1) = \frac{1}{3}[n(n + 1)(n + 2)]$.
14. By using mathematical induction prove that the given equation is true for all positive integers.

$2 + 4 + 6 + ... + 2n = n(n+1)$

15. Using the principle of mathematical induction, prove that

$1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + ... + (2n - 1)(2n + 1) = \frac{1}{3}[n(4n^2 + 6n - 1)]$

16. By using mathematical induction prove that the given equation is true for all positive integers.

$2 + 6 + 10 + ... + (4n - 2) = 2n^2$

17. Using the principle of mathematical induction, prove that $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

18. Using the principle of mathematical induction, prove that $\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \cdots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$

19. By induction prove that $3^n - 1$ is divisible by 2 is true for all positive integers.

20. Using the principle of mathematical induction, prove that

$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \cdots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$ for all n $\in$ N

21. By induction prove that $n^2 - 3n + 4$ is even and it is true for all positive integers.

22. Using the Principle of mathematical induction, prove that $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \ldots \left(1 - \frac{1}{n+1}\right) = \frac{1}{n+1}$